P425/2

APPLIED MATHEMATICS

PAPER 2

JUNE 2015

3 HOURS

EXTERNAL MOCK EXAMINATIONS-SET ONE 2015

UGANDA ADVANCED CERTIFICATE OF EDUCATION

APPLIED MATHEMATICS

PAPER 2

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in Section **A** and any **Five** from Section **B.***

*All necessary working **must** be shown clearly.*

Begin each answer on a fresh page.

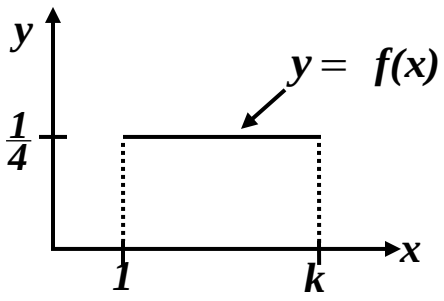
*In numerical work**,** take **g** to be* $\mathbf{9 \cdot 8 ms^{-2}}$.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

*Answer **all** the questions in this section.*

1. A uniformly distributed **r·v** ***X*** has the following p·d·f ***f (x):***



Find the**:**

(i) value of ***k*** ***(02 marks)***

(ii) equations of the p·d·f of ***X*** ***(01 mark)***

(iii) variance of ***X*** ***(02 marks)***

2. The cumulative distribution of the weights in kg of **50** students were as follows**:**

Weights (kg)	***< 20***	***< 30***	***< 40***	***< 50***	***< 60***	***< 70***
Cumulative Frequency	***0***	***8***	***20***	***34***	***44***	***50***

Calculate the**:**

(i) mean mark ***(03 marks)***

(ii) 60^{th} percentile ***(02 marks)***

3. An elastic string of natural length ***0·6m*** is stretched ***8cm*** by a hanging particle of mass ***2kg.*** Find the**:**

(i) elastic modulus of the string ***(03 marks)***

(ii) energy stored in the string when its length is ***0·7m*** ***(02 marks)***

4. Show that the equation $\boldsymbol{tanx - 4x = 0}$ has a root between **1·3** and **1·4.** Hence use linear interpolation method to find the root correct to ***three*** significant figures**.** ***(05 marks)***

5. Events **A** and **B** are such that $\boldsymbol{P(A) = \frac{8}{15}}$, $\boldsymbol{P(B) = \frac{1}{3}}$ and $\boldsymbol{P(A/B) = \frac{1}{5}}$**.**

Find**:**

(i) ***P(A U B)***

(ii) ***P(A or B but not A and B)*** ***(05 marks)***

6. A Particle ***P*** moves along the ***x- axis.*** The velocity of ***P*** at time ***t*** seconds is given by $\boldsymbol{v = 2t^2 - 14t + 20\ ms^{-1}}$**.** Find the**:**

(i) times when ***P*** is instantaneously at rest ***(03 marks)***

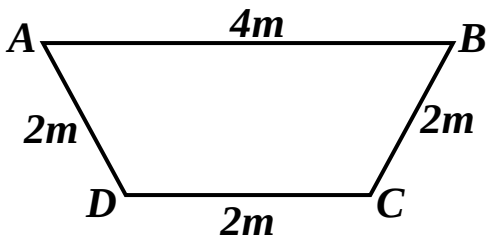
(ii) time taken for ***P*** to attain its greatest speed ***(02 marks)***

7. The base and height of a triangle were measured as ***5cm*** and ***6cm*** with relative errors ***0·04*** and ***0·05*** respectively. Find the**:**

(i) absolute error in each dimension ***(02 marks)***

(ii) range within which the area of the triangle lies. ***(03 marks)***

8. An isosceles trapezium ***ABCD*** is a uniform lamina with ***AB = 4m*** and ***BC = CD = DA = 2m*** *as shown**:***



Show that the distance of the centre of gravity of the lamina is $\frac{4\sqrt{3}}{9}m$ from ***AB*** ***(05 marks)***

SECTION B: (60 MARKS)

*Answer any **five** questions from this section. All questions carry equal marks.*

9. (i) By drawing graphs of $y = x$ and $y = In(8 - x)$ on the same axes, find the initial approximate root of the equation $x - In\,(8 - x) = 0$.

(06 marks)

(ii) Use Newton Raphson's method to find the root of the equation in **(i)** above correct to **4** significant figures. ***(06 marks)***

10. The marks of **40** students in a test were as follows**:**

marks	***30 –***	***40 –***	***50 –***	***60 –***	***70 –***	***80 –***
frequency	***8***	***5***	***12***	***9***	***6***	***0***

(a) Calculate the**:**

(i) mean mark ***(03 marks)***

(ii) standard deviation ***(03 marks)***

(b) Draw an ogive for the data and use it to estimate the number of students whose marks lie within one standard deviation of the mean ***(06 marks)***

11. The distribution function of a continuous **r·v** X is as follows**:**

$$\mathbf{F}(x) = \begin{cases} 0 & , \ x < 1 \\ \beta(x-1)^2 & , \ 1 \le x \le 3 \\ 1 & , \ x > 3 \end{cases}$$

Find**:**

(i) the value of β ***(02 marks)***

(ii) $\mathbf{P(1 \cdot 5 \le X \le 2)}$ ***(02 marks)***

(iii) the median of X ***(03 marks)***

(iv) the p·d·f of X ***(02 marks)***

(v) the mean of X ***(03 marks)***

12. (a) A Particle is projected from a point ***O*** with speed ***u*** at an angle of elevation θ to the horizontal**.** Show that its height ***y*** above ***O*** when it has travelled a distance ***x*** horizontally is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}.$$ ***(04 marks)***

(b) After ***2s*** of projection**,** a particle projected from the top of a vertical cliff ***6m*** high with speed ***u*** at an angle of elevation θ to the horizontal passes just above the top of a vertical post which is ***4m*** high and ***8m*** horizontally away from the base of the cliff**.**

(i) Show that $\tan\theta = 2{\cdot}2$ ***(06 marks)***

(ii) Find the value of ***u*** ***(02 marks)***

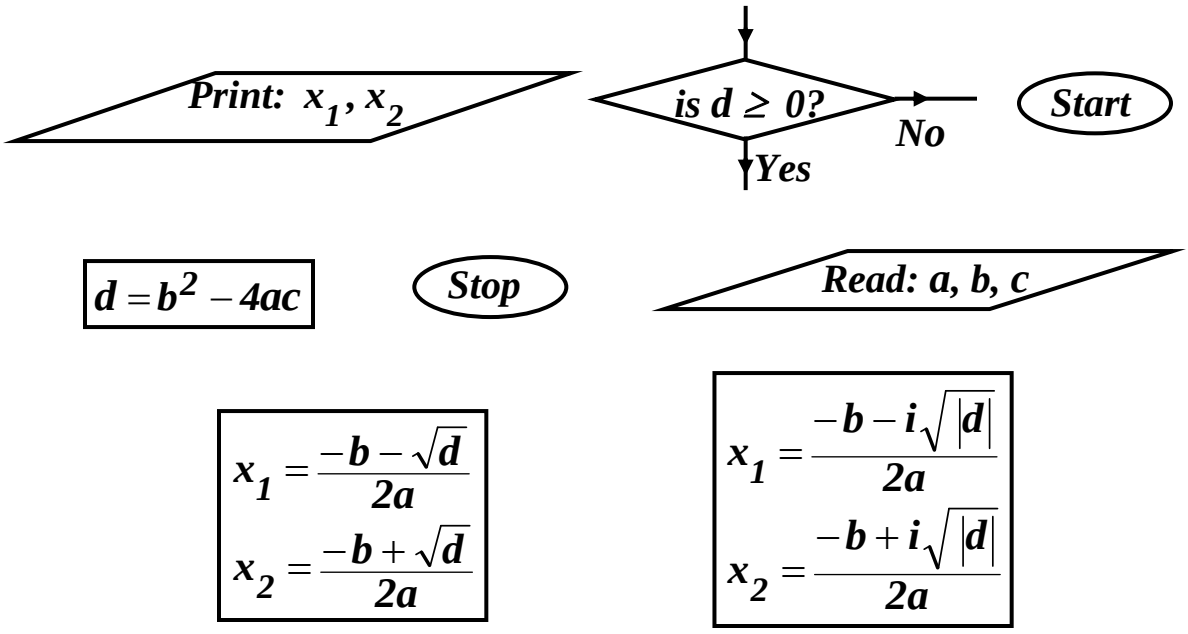
13. (a) A certain function ***f(x)*** has the following values**:**

x	***1***	***1·2***	***1·4***	***1·6***	***1·8***	***2***
f(x)	***1·4142***	***1·4832***	***1·5492***	***1·6125***	***1·6733***	***1·7321***

Use the trapezium rule to estimate $\int_1^2 f(x)\,dx$ correct to **3** decimal places**.** Suggest how the accuracy of this result may be improved.

(04 marks)

(b) The quadratic formula method for solving the equation $ax^2 + bx + c = 0$ is described by the following parts of the flowchart**:**



(i) By rearranging the given parts, draw a flow chart that shows the algorithm for the described method ***(05 marks)***

(ii) By performing a dry run for the flow chart using $x^2 - 4x + 13 = 0$, copy and complete the table below**:**

D	x_1	x_2
- - - -	- - - -	- - - -

(03 mark)

14. A non uniform ladder ***PQ*** of length ***12m*** and weight ***W*** has its centre of gravity at the point of trisection of its length**,** nearer to ***P.*** The ladder rests in limiting equilibrium with its end ***Q*** against a rough vertical wall and end ***P*** on a rough horizontal ground with which the coefficients of friction at each end being $\frac{1}{5}$ and $\frac{1}{3}$ respectively. Find**:**

(i) the angle of inclination θ of the ladder to the horizontal

(ii) how far up the ladder can a man of weight $5W$ climb before it slips**.** ***(12 marks)***

15. (a) The chances of three soldiers ***A, B*** and ***C*** hitting a bullet at a target are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. If all of them shoot once, find the probability of hitting the target by**:**

(i) all the soldiers**.** ***(02 marks)***

(ii) only two soldiers**.** ***(04 marks)***

(b) A student answers ***3*** questions**.** The chance of getting each question correct is $\frac{2}{3}$**.** Find the**:**

(i) probability distribution for the number of correct answers

(ii) probability of obtaining at most ***2*** correct answers**.** ***(06 marks)***

16. A car of mass **1000kg** is driven by an engine working at constant power at all speeds against a resistance to motion which is proportional to its speed**.** It can travel at a maximum speed of $25ms^{-1}$ up a plane inclined at $sin^{-1}\left(\frac{1}{7}\right)$ to the horizontal and its acceleration down the same plane is $1 \cdot 44ms^{-2}$ at the instant when its speed is $35ms^{-1}$**.** Find the**:**

(i) power of the engine**.** ***(10 marks)***

(ii) non gravitational resistance to motion at the instant when the speed of the car is $30ms^{-1}$**.** ***(02 marks)***

END