## THE RECTANGULAR OR UNIFROM DISTRIBUTION

A continuous random variable X having a p.d.f, f(x) where;

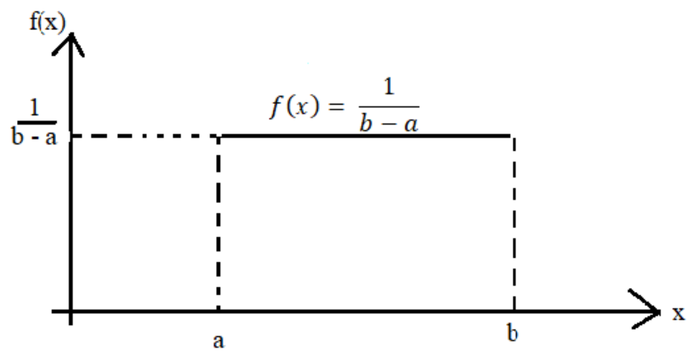
$$f(x) = \frac{1}{b-a}, for\ a \le x \le b$$

Where a and b are constants is said to follow a rectangular(uniform) distribution.

Note: a and b are parameters of the distribution

Hence $X \sim R(a, b)$

**Graph of f(x)**




X is random variable, since

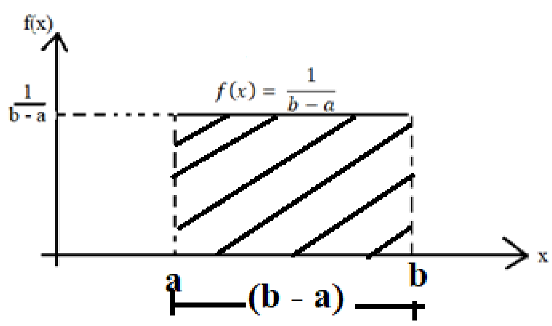
$$\int_{all\ x} f(x)dx = \int_a^b \frac{1}{(b-a)} dx$$

$$= \frac{1}{(b-a)}[x]_a^b$$

$$= \frac{1}{(b-a)}(b-a)$$

$$= 1$$

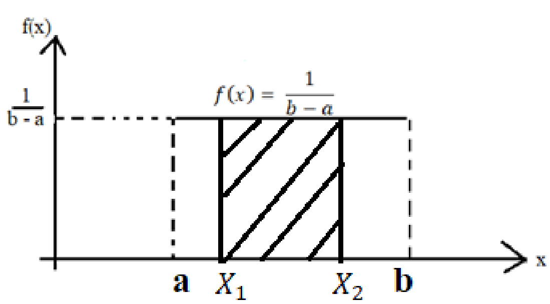
Alternatively;




$$Total\ area\ under\ the\ graph = 1$$

$$\frac{1}{(b-a)} x(b-a) = 1$$

Finding Probability $P(X_1 \le x \le X_2)$




$$P(X_1 \le x \le X_2) = \int_{X_1}^{X_2} f(x)dx$$

Or

$$P(X_1 \le x \le X_2) = Area\ of\ shaded\ part$$

$$= \frac{1}{(b-a)}(X_1 - X_2)$$

## Examples

1. If the continuous r.v X is such that $X \sim R(3,6)$. Find;
   (a) the p.d.f of X
   (b) P(X>5)

**Solution**

(a) $f(x) = \begin{cases} \frac{1}{6-3} & ; 3 \le x \le 6 \\ 0 & elsewhere \end{cases}$

$= \begin{cases} \frac{1}{3} & ; 3 \le x \le 6 \\ 0 & elsewhere \end{cases}$

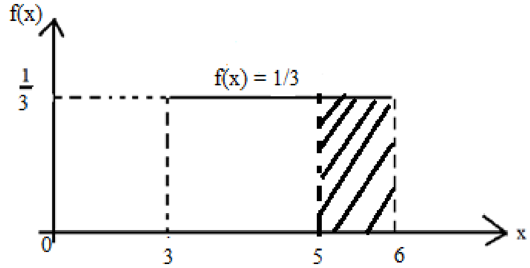
(b) $P(X > 5) = \int_5^6 \frac{1}{3} dx$

$= \left[\frac{x}{3}\right]\Big|_5^6$

$= \frac{(6-5)}{3}$

$= \frac{1}{3}$

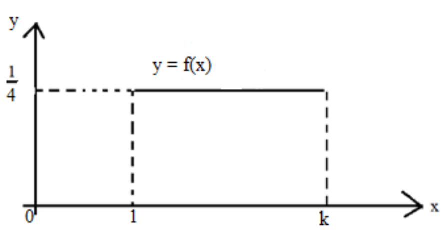
OR




$P(X > 5) = Area\ of\ shaded\ part$

$= \mathbf{\frac{1}{3} x (6-5) = \frac{1}{3}}$

2. The continuous r.v X has a p.d.f, f(x) as shown in the diagram below;




Find (a) the value of k

(b) P(2.5 < X < 3.2)

**Solution**

$(a)$ $Total\ area\ under\ the\ graph = 1$

$\frac{1}{4} x (k-1) = 1 \quad \rightarrow k = 5$

$(b)$ $P(2.5 < X < 3.2) = \frac{1}{4} x (3.2 - 2.5)$ ***OR*** $P(2.5 < X < 3.2) = \int_{2.5}^{3.2} \frac{1}{4} dx = \left[\frac{x}{4}\right]_{2.5}^{3.2}$

$= \frac{(3.2-2.5)}{4} = \mathbf{0.1750}$



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## EXPECTATION AND VARIANCE

3. The random variable X has a p.d.f given by;

$$f(x) = \begin{cases} \frac{1}{b-a} & for\ a \le x \le b \\ 0 & elsewhere \end{cases}$$

(a) Show that the mean is $\frac{(b+a)}{2}$ and the variance is $\frac{(b-a)^2}{12}$ for this distribution

(b) Given that the mean equals 1 and the variance equals $^4/_3$, find;

(i) $P(X < 0)$

(ii) The value of z such that $P(X > z + \sigma) = \frac{1}{4}$

**Solution**

(a) Mean (Expectation), $E(\mathrm{X}) = \int_a^b \mathrm{x.f(x)dx}$

$$= \int_a^b \mathrm{x.}\left(\frac{1}{\mathrm{b-a}}\right)\mathrm{dx}$$

$$= \frac{1}{\mathrm{b-a}}\left[\frac{\mathrm{x}^2}{2}\right]_a^b$$

$$= \frac{1}{\mathrm{b-a}}\left(\frac{\mathrm{b}^2-\mathrm{a}^2}{2}\right)$$

$$= \frac{1}{\mathrm{(b-a)}} \mathrm{x} \frac{\mathrm{(b+a)(b-a)}}{2}$$

$$= \frac{\mathrm{(b+a)}}{2}$$

Variance, $Var(X) = E(X^2) - E^2(X)$

But $E(X^2) = \int_a^b \mathrm{x}^2.\mathrm{f(x)dx} = \int_a^b \mathrm{x}^2.\left(\frac{1}{\mathrm{b-a}}\right)\mathrm{dx} = \frac{1}{\mathrm{b-a}}\left[\frac{\mathrm{x}^3}{3}\right]_a^b$

$$= \frac{1}{\mathrm{b-a}}\left(\frac{\mathrm{b}^3-\mathrm{a}^3}{3}\right)$$

$$= \frac{1}{3\mathrm{(b-a)}}(b-a)(b^2+ab+a^2)$$

$$= \frac{(\mathbf{b^2+ab+a^2})}{\mathbf{3}}$$


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Hence; $Var(X) = \frac{(b^2+ab+a^2)}{3} - \left(\frac{b+a}{2}\right)^2$

$$= \frac{(b^2+ab+a^2)}{3} - \left(\frac{b^2+2ab+a^2}{4}\right)$$

$$= \frac{4b^2+4ab+4a^2) - (3b^2+6ab+3a^2)}{12}$$

$$= \frac{(b^2-2ab+a^2)}{12}$$

$$= \frac{(b-a)^2}{12}$$

(b) Mean, $\frac{b+a}{2} = 1 \quad \therefore b + a = 2 \ldots\ldots\ldots..(1)$

Variance $\frac{(\boldsymbol{b}-\boldsymbol{a})^2}{\mathbf{12}} = \frac{\mathbf{4}}{\mathbf{3}} \quad \rightarrow (b-a)^2 = 16$

$\therefore (b-a) = 4 \ldots\ldots..(2)$

Adding (1) and (2); $2b = 6 \quad \rightarrow b = 3$
From eqn(1) $a = 2 - 3 = -1$

$$\therefore f(x) = \begin{cases} \frac{1}{4} & -1 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

(i) $P(X < 0) = \int_{-1}^{0} \frac{1}{4} dx = \frac{1}{4}[x]_{-1}^{0} = \frac{1}{4}(0 - -1) = \frac{\mathbf{1}}{\mathbf{4}}\ \boldsymbol{or}\ \mathbf{0.25}$

(ii) $P(X > z + \sigma) = \frac{1}{4}$ But $\sigma = \sqrt{var(x)} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$

Hence $\int_{z+\frac{2}{\sqrt{3}}}^{3} \left(\frac{1}{4}\right) dx = \frac{1}{4}$

$$\frac{1}{4}[x]_{z+\frac{2}{\sqrt{3}}}^{3} = \frac{1}{4}$$

$$\frac{1}{4}\left[3 - (z + \frac{2}{\sqrt{3}})\right] = \frac{1}{4} \qquad \rightarrow z = 2 - \frac{2}{\sqrt{3}}$$

$$3 - \left(z + \frac{2}{\sqrt{3}}\right) = 1$$

$\therefore \mathbf{z = 0.8453}$



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## EXERCISE

1. A r.v takes values k such that $0 \leq k \leq 5$ and is rectangularly distributed in the interval. Find the;
   (a) mean and standard deviation of k
   (b) $P(k < 3/k > 1)$

2. A random variable X has a probability density function (p.d.f) given by;

$$f(x) = \begin{cases} \frac{1}{k} & ; n \leq x \leq m \\ 0 & \text{elsewhere.} \end{cases} \quad \text{Where } k = m - n$$

   a) Show that the variance of x is given by $\frac{(m-n)^2}{12}$

   b) In preparation of Club handover party at GHS, the number of guests takes on a random variable X with a uniform distribution over the interval **(p, q)**. The expected number of guests is 9 with variance 12. Determine the;
      (i) values of **p** and **q**
      (ii) probability that at most 10 guests will attend.

3. If the continuous r.v X has a p.d.f, f(x) wehere f(x) = k and X$\sim R(-5,-2)$. Find;
   (a) The value of the constant k
   (b) E(X)
   (c) Var(X)
   (d) P(-4.3<X<-2.8)

4. The daily number of patients visiting a certain hospital in uniformly distributed between 150 and 210
   (a) Write down the probability distribution function (p.d.f) of the number of patients
   (b) Find the probability that between 170 and 14 patients visit the hospital on a particular day


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