

**GAYAZA HIGH SCHOOL**

**Uganda Advanced Certificate of Education**

**APPLIED MATHEMATICS**

**Paper 2**

3 Hours.

**INSTRUCTIONS TO CANDIDATES:**

Attempt **all** the **eight** questions in Section **A** and **five** from section **B**.

Begin each answer on a fresh sheet of paper

Mathematical tables with a list of formulae and squared papers are provided.

Silent non-programmable scientific calculators may be used.

In numerical work, take  $g = 9.8 \text{ m s}^{-2}$ .

State the degree of accuracy at the end of each question attempted using a calculator or tables indicate **Cal** for calculator, or **Tab** for mathematical tables.

## SECTION A: (40 MARKS)

Attempt **all** the questions in this section.

- Two particles P and Q are moving along a straight path. When Q is ahead of P by  $14m$  the speed of Q is  $16 m s^{-1}$  and that of P is  $25 m s^{-1}$ . Given that P and Q have a constant retardation of  $6 m s^{-2}$  and  $4 m s^{-2}$  respectively, find the distance Q has travelled when it is first overtaken by P.
- Which of the following is a better iterative formula that can be used to find the root of  $2x^3 + 5x - 8 = 0$  to 1 d.p taking  $x_0 = 1$ .  
$$x_{n+1} = \frac{2}{5} \left( 4 - (x_n)^3 \right) \quad ; \quad x_{n+1} = \left( 4 - \frac{5}{2} x_n \right)^{\frac{1}{3}}$$
- The probability that a married man watches a football match on television is 0.4 and the probability that a married woman watches the same match is 0.5. The probability that the man watches the match, given that his wife does, is 0.7. Find:
  - The probability that a married couple watches the football match
  - The probability that a wife watches the match given that her husband does.
- The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that between 84 and 95 inclusive of the next 100 patients having this operation survive?
- A particle is projected from a point P with an initial speed of  $78.4 m s^{-1}$  at an angle of  $30^\circ$  with the horizontal. Calculate the;
  - velocity of the particle  $\frac{1}{2}$  second after leaving P.
  - greatest height reached.
- In order to determine whether or not there is any correlation between secondary school pupils' performances in mathematics and English two standard tests (one in mathematics and the other in English) were given to 12 pupils. The following marks were recorded:

Pupils	A	B	C	D	E	F	G	H	I	J	K	L
Marks in Mathematics	72	65	85	89	75	65	67	43	46	51	70	77
Marks in English	65	65	75	74	76	50	52	47	46	53	65	69

Calculate the rank correlation coefficient for the pupils' performance in the two subjects.

7. Forces of 2N, 4N, 3N and 2N act along the sides PQ, QR, RS, SP, of a square PQRS of side 3m. A force of  $\sqrt{5}N$  acts along QM where M is the mid-point of PS. The line of action of their resultant cuts PQ produced at K. Calculate
- The magnitude of this resultant
  - The angle this resultant makes with PQ.
8. Triangle ABC has side  $AB = 3.7$  cm,  $BC = 5.7$  cm both values have been measured to 2 significant figures and angle at B is  $120^\circ \pm 0.50^\circ$ . Find the limits within which the area of the triangle lies.

### SECTION B (60 MARKS)

Attempt any **five** questions from this section. All questions carry equal marks.

- 9a) The mass of a cheese biscuit has a normal distribution with mean 6g and standard deviation 0.2g. The mass of a ginger biscuit also has a normal distribution with mean 10g and standard deviation 0.3g. Determine the probability that a collection of seven cheese biscuits has a mass greater than a collection of four ginger biscuits.

- b) From a random sample of 576 of a company's 20,000 employees, it is found that the average number of days each person was absent from work due to illness was eight days a year, with a standard deviation of 3.6 days. Find the 99% confidence limits for the average number of day's absence a year through sickness per employee for the company as a whole.

- 10i) Find an approximate value of  $\int_0^{\frac{1}{2}} \sqrt{1-x^2} dx$  to 3d.p by using the trapezium rule with 6 ordinates.

- ii) Find the exact value of  $\int_0^{\frac{1}{2}} \sqrt{1-x^2} dx$ .

- iii) Find the percentage error in the calculations (i) and (ii) above. How can this error be reduced.

11. A random variable  $X$  has a probability density function  $f(x)$  given by

$$f(x) = \begin{cases} Cx & 0 \leq x \leq 1 \\ C & 1 \leq x \leq 2 \\ C(3-x) & 2 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

- a) Sketch the graph of the function above

- b) Find the: i) value of  $C$

- ii) mean of  $X$

iii)  $P \left( \left( \frac{1}{2} \leq x \leq \frac{5}{2} \right) \right) / \left( \left( x \geq \frac{3}{2} \right) \right)$

12. Use a graphical method to find the number of real roots of the equation  $x^4 + x - 3 = 0$ . By using the Newton-Raphson formula; find an approximate value of the positive root of the equation to 2d.p.

- 13a) A particle of  $1\text{ kg}$  is moving under the action of a constant force  $\mathbf{F} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}\text{ N}$ . Find the work done by  $\mathbf{F}$  when the particle moves from the point A with position vector  $5\mathbf{j} + 2\mathbf{k}$  to the point B with position vector  $3\mathbf{i} + \mathbf{k}$ . If the speed of the particle at A is  $6\text{ m s}^{-1}$ , find the speed of the particle at B.
- b) A small pulley is fixed at a height of  $3\text{m}$  vertically above a point K of a horizontal rough table. A body of mass  $6.5\text{kg}$  rests on the table at a distance  $4\text{m}$  from K. A string, with one end fastened to the body, passes over the pulley and has a mass of  $2.5\text{kg}$  hanging from the other end. Given that the body is on the point of slipping, calculate the coefficient of friction between the body and the table.
- 14a) A maintenance manager has recorded the time taken by maintenance staff to repair a particular type of equipment fault. The following table shows the time taken in minutes to repair 250 recent faults.

Time taken (Minutes)	Frequency
Under 10	28
10 and under 20	54
20 and under 30	81
30 and under 40	57
40 and under 50	23
50 and under 60	7
60 and over	0

- a) Calculate the mean and the standard deviation of the time taken.

- b) Plot an Ogive and use it to estimate;
- (i) the median time
  - (ii) number of faults repaired between 27 and 42 minutes.

15a) To a man cycling due North at  $12\text{kmh}^{-1}$  the wind appears to be blowing from a direction  $030^{\circ}$ , whilst to a man walking due East at  $4\text{kmh}^{-1}$  the wind appears to be blowing from a direction  $060^{\circ}$ . Find the direction of the wind.

b) A ship A is travelling on a course of  $060^{\circ}$  at a speed of  $30\sqrt{3}\text{ kmh}^{-1}$  and a ship B is travelling on a course of  $030^{\circ}$  at  $20\text{kmh}^{-1}$ . If at noon B is 260km due east of A, show that if the ships maintain their velocities collision is not expected.

16a) A particle of mass  $4\text{ kg}$  is acted upon at time  $t$  by a force  $\mathbf{F}$ , where  $\mathbf{F} = 8\mathbf{i} - 48t^2\mathbf{j} + 72t\mathbf{k}$ . At time  $t = 0$  the particle is at the point with position vector  $\mathbf{i} - 5\mathbf{j} - \mathbf{k}$  and its velocity is  $3(\mathbf{i} + \mathbf{j})$ . Find the position vector of the particle at time  $t = 2$ .

b) A uniform ladder, of weight  $W$  and length  $l$ , rests with one end against a smooth vertical wall. The other end of the ladder rests on a rough horizontal ground, the coefficient of friction between the ladder and the ground being  $\frac{1}{4}$ . The ladder is inclined at an angle

$\theta$  where  $\tan \theta = \frac{24}{7}$ . A man of weight  $10W$  climbs up the ladder. Show that the man can reach

a height of  $\frac{6l}{7}$  above the ground before the ladder begins to slip.

**END**