1.	Solve the equation: $\tan 4x + \tan 2x = 0$ for $0^o \le x \le 2\pi$ $\frac{2\tan 2x}{1-\tan^2 2x} + \tan 2x = 0$, $2\tan 2x + \tan 2x - \tan^3 2x = 0$ $\tan 2x(3-\tan^2 2x) = 0$ $\tan 2x = 0,\ 2x = 0^o, 180^o, 360^o$, $x = 0^o, \frac{\pi}{2}, \pi$ $\tan 2x = \pm\sqrt{3},\ 2x = 60^o, 120^o, 240^o, 300^o$, $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2}{3}\pi, \frac{5}{6}\pi$ ALTERNATIVELY $\frac{\sin 4x}{\cos 4x} + \frac{\sin 2x}{\cos 2x} = 0$ $\sin 4x \cos 2x + \cos 4x \sin 2x = 0$ $\sin(4x + 2x) = \sin 6x = 0$ $6x = 0^o, 180^o, 360^o, 540^o, 720^o, 900^o, 1080^o$ $x = 0^o, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2}{3}\pi, \frac{5}{6}\pi, \pi$	
2.	Find the coordinates of the circumscribing circle which passes through the points $A(1,\ 2)$, $B(2,\ 5)$ and $C(-3,\ 4)$. Perpendicular bisectors of two chords intersect at the centre of the circle. Midpoint of $AB = \left(\frac{3}{2}, \frac{7}{2}\right)$, mid point $BC = \left(-\frac{1}{2}, \frac{9}{2}\right)$ Gradient of $AB = \frac{5-2}{2-1} = 3$, Gradient of $BC = \frac{4-5}{-3-2} = \frac{-1}{-5} = \frac{1}{5}$ Gradient of normal to $AB = \frac{-1}{3}$, that to $BC = -5$	

	Equation through $\left(\frac{3}{2}, \frac{7}{2}\right)$ is $\frac{y-\frac{7}{2}}{x-\frac{3}{2}} = \frac{-1}{3}$, to get $3y + x = 8 \ldots$(i) Equation through $\left(-\frac{1}{2}, \frac{9}{2}\right)$ is $\frac{y-\frac{9}{2}}{x+\frac{1}{2}} = -5$ to get $y + 5x = 2$..(ii) Solve eqn(i) and eqn(ii) to get $x = -\frac{3}{7}$ and $y = \frac{29}{7}$ Thus centre is $C\left(-\frac{3}{7}, \frac{29}{7}\right)$ ALT: The general equation can be $x^2 + y^2 - 2gx - 2fy + c = 0$ $A(1,\ 2)$; $1+4-2g-4f+c=0$, $5-2g-4f+c=0$ ……..(i) $B(2,\ 5)$; $4+25-4g-10f+c=0$ $29-4g-10f+c=0$ …..(ii) $C(-3,\ 4)$; $9+16+6g-8f+c=0$ $25+6g-8f+c=0$ …..(iii) Eqn(ii) – eqn(i): $2g+6f=24$, $g+3f=12$ Eqn(iii) –eqn(ii): $5g+f=2$, Solving: $g=-\frac{3}{7}$, $f=\frac{29}{7}$, $c=\frac{75}{7}$ Equation is; $x^2 + y^2 + \frac{6}{7}x - \frac{58}{7}y + \frac{75}{7} = 0$, Thus centre is $C\left(-\frac{3}{7}, \frac{29}{7}\right)$	
3.	If $y = e^{-x}\cos x$, prove that $\frac{dy}{dx} = -\sqrt{2}e^{-x}\cos\left(x - \frac{\pi}{4}\right)$. $y = e^{-x}\cos x$, $\frac{dy}{dx} = -e^{-x}(\cos x + \sin x)$, for t.p $\frac{dy}{dx} = 0$ $-e^{-x}(\cos x + \sin x) = 0$, for $-e^{-x} \neq 0$	

	$(\cos x+\sin x)=0$, let $(\cos x+\sin x)\equiv R\cos(x-\alpha)\equiv R\cos x\cos\alpha+R\sin x\sin\alpha$ $R\cos\alpha\equiv 1$, $R\sin\alpha\equiv 1$ thus $\tan\alpha=1$, $\alpha=\frac{\pi}{4}$ and $R=\sqrt{2}$ Therefore $\frac{dy}{dx}=-\sqrt{2}e^{-x}\cos\left(x-\frac{\pi}{4}\right)$ as required.	
4.	Find the perpendicular distance from the point $A(1,\ 2,\ -4)$ to the plane which passes through the point $B(1,\ 4,\ 9)$ and is normal to the vector $3\mathbf{i}-\mathbf{k}$. Equation of the plane $\mathbf{r}.\begin{pmatrix}3\\0\\-1\end{pmatrix}=\begin{pmatrix}1\\4\\9\end{pmatrix}.\begin{pmatrix}3\\0\\-1\end{pmatrix}$ $3x-z=-6$ OR $3x-z+6=0$ $\Rightarrow$ perpendicular distance $d=\frac{3\times1-1\times-4+6}{\sqrt{9+1}}=\frac{13}{\sqrt{10}}=\frac{13}{10}\sqrt{10}$	
5.	Given that $y=In\left(x+\sqrt{x^2+a^2}\right)$, where a is a constant, prove that $\frac{dy}{dx}=\frac{1}{\sqrt{x^2+a^2}}$ and hence evaluate $\int_0^4\frac{dx}{\sqrt{x^2+9}}$. $y=In\left(x+\sqrt{x^2+a^2}\right)$, $\frac{dy}{dx}=\frac{1+\frac{2x}{2\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$ $\frac{dy}{dx}=\frac{\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$ $\therefore\ \frac{dy}{dx}=\frac{1}{\sqrt{x^2+a^2}}$ $\int_0^4\frac{dx}{\sqrt{x^2+9}}$, since $\frac{d}{dx}\left(In\left(x+\sqrt{x^2+a^2}\right)\right)=\frac{1}{\sqrt{x^2+a^2}}$ $\Rightarrow\int_0^4\frac{dx}{\sqrt{x^2+9}}=\left[In\left(x+\sqrt{x^2+9}\right)\right]_0^4=(In9-In3)=In3$	

6.	If $z = 1 + 2i$ is a root of the equation $z^3 + az + b = 0$ where a and b are real, find the values of a and b. $(1+2i)^3 + a(1+2i) + b = 0$ $-11 - 2i + a + 2ai + b = 0$, $(-11 + a + b) + (2a - 2)i = 0$ Thus, $2a - 2 = 0$, $a = 1$ $(-11 + 1 + b) = 0$, $b = 10$	
7.	Solve the equations: $\frac{x^2}{y} + \frac{y^2}{x} = 9$, $x + y = 6$ From eqn (1) we have $x^3 + y^3 = 9xy$, thus $(x+y)(x^2 - xy + y^2) = 9xy$ Thus $6(x^2 - xy + y^2) = 9xy$ $2x^2 - 5xy + 2y^2 = 0$, $x + y = 6$ so, $x = 6 - y$ $2(6-y)^2 - 5y(6-y) + 2y^2 = 0$ to get $9y^2 - 54y + 72 = 0$ $y^2 - 6y + 8 = 0$ $(y-4)(y-2) = 0$ so, $y = 2, 4$ and $x = 4, 2$	
8.	Evaluate: $\int_0^{1/4} \cos^{-1} 2x \, dx$ $\int_0^{1/4} \cos^{-1} 2x \, dx$ Let $u = \cos^{-1} 2x$, $\frac{du}{dx} = \frac{-2}{\sqrt{1-4x^2}}$ $\frac{dv}{dx} = 1$, $v = x$ $= \left[x \cos^{-1} 2x\right]_0^{1/4} - \int_0^{1/4} \frac{-2x}{(1-4x^2)^{1/2}} dx$ $= \left[x \cos^{-1} 2x\right]_0^{1/4} - \frac{1}{2}\left[(1-4x^2)^{1/2}\right]_0^{1/4}$	

	$=\left(\frac{1}{4}.\frac{\pi}{3}-0\right)-\frac{1}{2}\left(\frac{\sqrt{3}}{2}-1\right)$ $=\frac{\pi}{12}-\frac{\sqrt{3}}{4}+\frac{1}{2}\approx 0.3288$	
9a)	The remainder when the expression x^3-2x^2+ax+b is divided by $x-2$ is five times the remainder when the same expression is divided by $x-1$, and 12 less than the remainder when the same expression is divided by $x-3$. Find the values of a and b. $x^3-2x^2+ax+b\equiv(x-2)Q(x)+5R$ $x^3-2x^2+ax+b\equiv(x-1)Q(x)+R$ $x^3-2x^2+ax+b\equiv(x-3)Q(x)+5R+12$ When $x=2$, $2a+b-5R=0$, When $x=1$, $2a+b-R=1$, When $x=3$, $3a+b-5R=3$ Solve to get $a=3,\ b=-1$	
b)	Given that the first three terms in the expansion in ascending powers of x of $(1+x+x^2)^n$ are the same as the first three terms in the expansion of $\left(\frac{1+ax}{1-3ax}\right)^3$, find the value of a and n. $(1+x+x^2)^n=1+n(x+x^2)+\frac{n(n-1)(x+x^2)^2}{2!}+\ldots$ $=1+nx+nx^2+\frac{1}{2}n(n-1)x^2+\ldots$ $\left(\frac{1+ax}{1-3ax}\right)^3=(1+ax)^3(1-3ax)^{-3}$	

	$=\left(1+3ax+3a^2x^2+\ldots\right)\left(1+9ax+18a^2x^2+\ldots\right)$ $=1+9ax+54a^2x^2+3ax+27a^2x^2+3a^2x^2+\ldots$ $=1+12ax+84a^2x^2+\ldots$ Thus $1+nx+nx^2+\frac{1}{2}n(n-1)x^2+\ldots=1+12ax+84a^2x^2+\ldots$ Equating coefficients, we get $n=12a$ ....(i) $84a^2=n+\frac{n(n-1)}{2}$ ....(ii) $84a^2=12a+\frac{12a(12a-1)}{2}$, $84a^2=12a+72a^2-6a$ $12a^2-6a=0$, for $a\neq 0$, $a=\frac{1}{2}$ thus $n=6$	
10a)	Show that the lines $\mathbf{r}=(-2\mathbf{i}+5\mathbf{j}-11\mathbf{k})+\lambda(3\mathbf{i}+\mathbf{j}+3\mathbf{k})$, $\mathbf{r}=(8\mathbf{i}+9\mathbf{j})+t(4\mathbf{i}+2\mathbf{j}+5\mathbf{k})$ intersect, hence, find the position vector of their point of intersection. Find also the Cartesian equation of the plane formed by these two lines. $\begin{pmatrix}-2\\5\\-11\end{pmatrix}+\begin{pmatrix}3\lambda\\\lambda\\3\lambda\end{pmatrix}=\begin{pmatrix}8\\9\\0\end{pmatrix}+\begin{pmatrix}4t\\2t\\5t\end{pmatrix}$ $\Rightarrow -2+3\lambda=8+4t$ .....(i) $5+\lambda=9+2t$ ...(ii) $-11+3\lambda=5t$ ...(iii) Eqn(i) – eqn(ii)x3 $\begin{array}{rcl}-2+3\lambda&=&8+4t\\-(15+3\lambda)&=&-(27+6t)\end{array}$ to get $-17=-19-2t$ $\Rightarrow t=-1$ then from eqn(i) $\lambda=2$	

	Substitute $t\ \&\ \lambda\ in\ (i)$, LHS. $-11+6=-5=$RHS Thus: position vector of point of intersection $\mathbf{r}=\begin{pmatrix}4\\7\\-5\end{pmatrix}$ Vector equation is given by $\mathbf{r}=\begin{pmatrix}4\\7\\-5\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\3\end{pmatrix}+t\begin{pmatrix}4\\2\\5\end{pmatrix}$ - $\Rightarrow\ x=4+3\lambda+4t\ldots$(i), $y=7+\lambda+2t\ldots$(ii) $z=-5+3\lambda+5t\ \ldots$(iii) Eqn(i) -3eqn(2) $\begin{matrix}x=4+3\lambda+4t\\ -\ 3y=-(21+3\lambda+6t)\end{matrix}$ to get $x-3y=-17-2t$ Thus $t=\frac{x-3y+17}{-2}=\frac{3y-x-17}{2}$ Put t in eqn(i); $x=4+3\lambda+2(3y-x-17)$ Thus $\lambda=\frac{3x-6y+30}{3}=x-2y+10$ Put t and λ in eqn(iii) to get: $z=-5+3(x-2y+10)+\frac{5}{2}(3y-x-17)$ $\therefore\ x+3y-2z-35=0$ ALTERNATIVE: $A(-2,,5,-11),\ B(8,9,0),\ C(4,7,-5)$ and let $p(x,y,z)$ $\vec{AB}=\begin{pmatrix}10\\4\\11\end{pmatrix},\ \vec{AC}=\begin{pmatrix}6\\2\\6\end{pmatrix},\ \vec{AP}=\begin{pmatrix}x+2\\y-5\\z+11\end{pmatrix}$	

	$\vec{AP} = \lambda\begin{pmatrix}10\\4\\11\end{pmatrix} + \mu\begin{pmatrix}6\\2\\6\end{pmatrix} = \begin{pmatrix}x+2\\y-5\\z+11\end{pmatrix} \Rightarrow \begin{matrix}x-10\lambda-6\mu=-2 \quad i\\ y-4\lambda-2\mu=5 \quad ii\\ z-11\lambda-6\mu=-11 \quad .....iii\end{matrix}$ Eqn(i) -3Eqn(ii) $\begin{matrix}x-10\lambda-6\mu=-2\\3y-12\lambda-6\mu=15\end{matrix}$ to get $x-3y+2\lambda=-17..........iv$ 3Eqn(ii) –Eqn(iii) $\begin{matrix}3y-12\lambda-6\mu=15\\z-11\lambda-6\mu=-11\end{matrix}$ to get $3y-z-\lambda=26..........v$ FromEqn(iv) and 2Eqn(v) $\begin{matrix}x-3y+2\lambda=-17\\6y-2z-2\lambda=52\end{matrix}$ to get $x+3y-2z-35=0$	
11a)	Differentiate with respect to x : i) $y=2x^{\cos x}$ $Iny=\cos x\, In2x$ $\frac{1}{y}\frac{dy}{dx}=-\sin x In2x+\frac{2\cos x}{2x}$ $\quad \frac{dy}{dx}=2x^{\cos x}(\cos x-\sin x In2x)$ ii) $y=\frac{e^{\sin x}}{\tan^{-1}x}$ $Iny=\sin x - In\tan^{-1}x$ $\frac{1}{y}\frac{dy}{dx}=\cos x-\frac{1}{(1+x^2)\tan^{-1}x}$ $\quad \frac{dy}{dx}=\frac{e^{\sin x}}{\tan^{-1}x}\left(\cos x-\frac{1}{(1+x^2)\tan^{-1}x}\right)$	
b)	Prove that $\int_1^3\left(\frac{3-x}{x-1}\right)^{1/2}dx=\pi$ (Use the substitution $x=3\sin^2\theta+\cos^2\theta$).	

	$\int_1^3 \left(\frac{3-3+2\cos^2\theta}{3-2\cos^2\theta-1}\right)^{1/2}.4\sin\theta\cos\theta.d\theta$ $x = 3-2\cos^2\theta$, $dx = 4\sin\theta\cos\theta\, d\theta$ $=\int_0^{\frac{\pi}{2}} \frac{\sqrt{2}\cos\theta}{\sqrt{2}\sin\theta}.4\sin\theta\cos\theta\, d\theta$ $x=1,\ \theta=0$; $x=3,\ \theta=\frac{\pi}{2}$ $=4\int_0^{\frac{\pi}{2}}\cos^2\theta\, d\theta \quad = 2\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)\, d\theta \quad = \quad 2\left[\left(\theta+\frac{1}{2}\sin 2\theta\right)\right]_0^{\frac{\pi}{2}}$ $= 2\left[\left(\frac{\pi}{2}-0\right)-(0)\right] \quad = \pi$	
12a	If A, B, C are angles of a triangle, prove that $\sin^2 A+\sin^2 B+\sin^2 C = 2+2\cos A\cos B\cos C$ From the L.H.S $\sin^2 A+\sin^2 B+\sin^2 C = \frac{1}{2}(1-\cos 2A)+\frac{1}{2}(1-\cos 2B)+\sin^2 C$ $= 1-\frac{1}{2}(\cos 2A-\cos 2B)+(1-\cos^2 C)$ $= 2-\frac{1}{2}(2\cos(A+B)\cos(A-B))-\cos^2 C$ $:\cos(A+B) = -\cos C$ $= 2-(-\cos C\cos(A-B))-\cos^2 C$ $= 2+\cos C(\cos(A-B)-\cos C)$ $= 2+\cos C(\cos(A+B)+\cos(A-B))$ $= 2+2\cos A\cos B\cos C$	
b)	By expressing $6\cos^2\theta+8\sin\theta\cos\theta$ in the form $R\cos(2\theta-\alpha)$, find the maximum and minimum values of the function and the corresponding value of θ, hence solve $6\cos^2\theta+8\sin\theta\cos\theta = 4$. $3(2\cos^2\theta)+4(2\sin\theta\cos\theta) = 3(1+\cos 2\theta)+4\sin 2\theta$	

	$= 3\cos 2\theta + 4\sin 2\theta + 3$ Let $3\cos 2\theta + 4\sin 2\theta + 3 \equiv R\cos 2\theta \cos\alpha + R\sin 2\theta \sin\alpha$ So, $R\cos 2\theta \equiv 3$, $R\sin 2\theta \equiv 4$ $\Rightarrow \tan\alpha = \frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{4}$ $\therefore \alpha = 53.1^o$ $R^2\cos^2\alpha + R^2\sin^2\alpha = 3^2 + 4^2$ $\therefore R = 5$ $\Rightarrow 3\cos 2\theta + 4\sin 2\theta + 3 = 5\cos(2\theta - 53.1^o) + 3$ Max. Value $=$ 8 when $2\theta - 53.1^o = 180^o$ $\therefore \theta = 116.55^o$ Min. Value $= -2$ when $2\theta - 53.1^o = 270^o$ $\therefore \theta = 161.55^o$ Thus: $6\cos^2\theta + 8\sin\theta\cos\theta = 4$ $5\cos(2\theta - 53.1^o) + 3 = 4$ $\therefore \cos(2\theta - 53.1^o) = \frac{1}{5}$ $2\theta - 53.1^o = 78.5^o, 281.5^o$ $\Rightarrow \theta = 65.8^o, 167.3^o$	
13.	The curve with the equation $y = \frac{ax+b}{x(x+2)}$ where a and b are constants has zero gradient at $(1, -2)$. (a) find the values of a and b. $y = \frac{ax+b}{x(x+2)}$, $\frac{dy}{dx} = \frac{a(x^2+2x) - (ax+b)(2x+2)}{(x^2+2x)^2}$ For turning points, $\frac{dy}{dx} = 0$ so when $x = 1$, $\frac{dy}{dx} = \frac{3a - 4(a+b)}{9} = 0$, so $a = -4b$ ....(i)	

Curve passes through $(1,\ -2)$, so $-2=\frac{a+b}{3}$, so $a+b=-6$.......(ii)

Thus $a=-8$ and $b=2$

(b) Find the equations for all the asymptotes to the curve, the turning points and sketch the curve.

$y=\frac{-8x+2}{x(x+2)}$, $\frac{dy}{dx}=\frac{-8(x^2+2x)-(-8x+2)(2x+2)}{(x^2+2x)^2}$ when $x=-\frac{1}{2}$

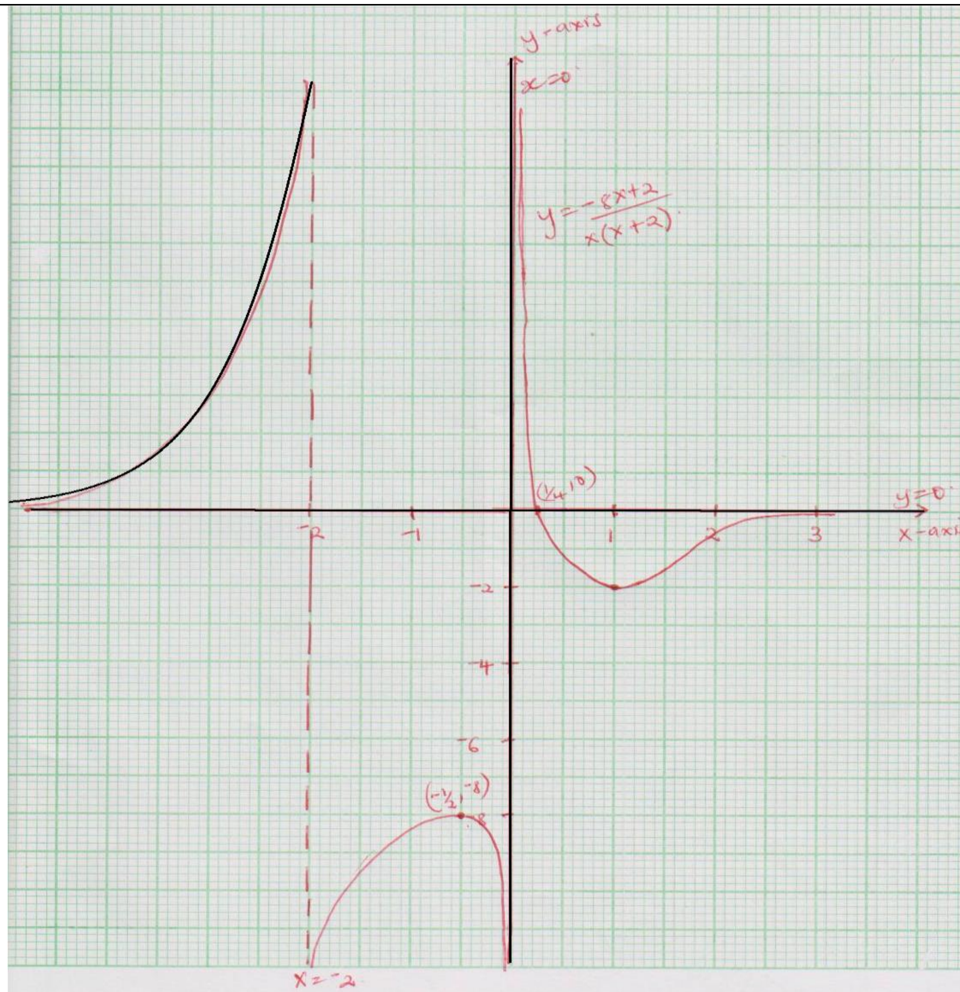
$$\frac{dy}{dx}=\frac{-8\left(\frac{1}{4}+-1\right)-(4+2)(-1+2)}{\left(\frac{1}{4}+-1\right)^2}=\frac{6-6}{\frac{9}{16}}=0 \text{ as required.}$$

Intercepts are: when $y=0$, $x=\frac{1}{4}$; $(\frac{1}{4},\ 0)$

Vertical asymptotes; $x=0,\ x=-2$

Hence the turning points are $(1,\ -2)$ and $(-\frac{1}{2},\ -8)$

	L	$x=1$	R	L	$x=-\frac{1}{2}$	R
Sign of $\frac{dy}{dx}$	-		+	+		-
		min			Max	



14a	Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. $\arg(z-3)-\arg(z-2i)=\frac{\pi}{4}$, for $z=x+iy$, $\arg((x-3)+iy)-\arg(x+i(y-2))=\frac{\pi}{4}$ $\tan^{-1}\left(\frac{y}{x-3}\right)-\tan^{-1}\left(\frac{y-2}{x}\right)=\frac{\pi}{4}$, let	

	$\tan^{-1}\left(\frac{y}{x-3}\right)=A,\quad \tan^{-1}\left(\frac{y-2}{x}\right)=B$ $\tan(A-B)=\tan\frac{\pi}{4}=1$, thus $\dfrac{\frac{y}{x-3}-\frac{y-2}{x}}{1+\left(\frac{y}{x-3}\right)\left(\frac{y-2}{x}\right)}=1$ $\frac{xy-xy+2x+3y-6}{x(x-3)}=\frac{x^2-3x+y^2-2y}{x(x-3)}$, $x^2+y^2-5x-5y+6=0$ $(x-\frac{5}{2})^2+(y-\frac{5}{2})^2=\frac{26}{4}$, thus the locus is a circle with the centre $\left(\frac{5}{2},\ \frac{5}{2}\right)$ and radius $\frac{\sqrt{26}}{2}$ units.	
b)	Find the four roots of: $-16i$. Let $z=-16i$, then $\lvert z\rvert=16$, $\arg z=\tan^{-1}-\frac{16}{0}=-\frac{\pi}{2}$ So $z=-16i=16\left(\cos-\frac{\pi}{2}+i\sin-\frac{\pi}{2}\right)$ $(-16i)^{1/4}=16^{1/4}\left(\cos\left(\frac{-\frac{\pi}{2}+2k\pi}{4}\right)+i\sin\left(\frac{-\frac{\pi}{2}+2k\pi}{4}\right)\right)$ for $k=0,1,2,3$ For $k=0$, $2\left(\cos\left(-\frac{\pi}{8}\right)+i\sin\left(-\frac{\pi}{8}\right)\right)=1.8478-0.7654i$ For $k=1$, $2\left(\cos\left(\frac{3\pi}{8}\right)+i\sin\left(\frac{3\pi}{8}\right)\right)=0.7654+1.8478i$ For $k=2$, $2\left(\cos\left(\frac{7\pi}{8}\right)+i\sin\left(\frac{7\pi}{8}\right)\right)=-1.8478+0.7654i$	

	For $k=3$, $2\left(\cos\frac{11\pi}{8}+i\sin\frac{11\pi}{8}\right)=-0.7654-1.8478i$	
15a	Find the equation of the tangent to the curve $y=x^3$ at the point $P(t,\ t^3)$. Prove that this tangent cuts the curve again at the point $Q(-2t,\ -8t^3)$ and find the locus of the mid point of PQ. $\frac{dy}{dx}=3x^2$, at $x=t$, the gradient of the tangent is $3t^2$ Thus the equation of the tangent at $P(t,\ t^3)$ is $\frac{y-t^3}{x-t}=3t^2$, to get $y=3t^2x-2t^3$. For the tangent to meet $y=x^3$ again, we solve simultaneously. $x^3=3t^2x-2t^3$, $x^3-3t^2x+2t^3=0$ But $x=t$ is a root. Use long division $x-t\overline{)x^3-3t^2x+2t^3}$ (quotient $x^2+xt-2t^2$) to get quotient as $x^2+xt-2t^2=0$, $(x-t)(x+2t)=0$ Thus the other point is when $x=-2t$ and $y=-8t^3$ Mid point of $PQ=\left(-\frac{t}{2},-\frac{7t^3}{2}\right)$, we are required to eliminate the parameter t to find the Cartesian equation. So for $x=-\frac{t}{2}$, $t=-2x$ and $y=-\frac{7t^3}{2}$, the locus is $y=28t^3$.	
b)	Given that the line $y=mx+c$ is a tangent to the circle $(x-a)^2+(y-b)^2=r^2$, show that $(1+m^2)r^2=(c-b+am)^2$. $y=mx+c$, $(x-a)^2+(y-b)^2=r^2$	

	$(x-a)^2+(mx+c-b)^2=r^2$ $x^2-2ax+a^2+m^2x^2+2m(c-b)x+(c-b)^2-r^2=0$ $(1+m^2)x^2+(2m(c-b)-2a)x+a^2+(c-b)^2-r^2=0$ For a tangent, $b^2-4ac=0$, $(2m(c-b)-2a)^2-4(1+m^2)(a^2+(c-b)^2-r^2)=0$, $m^2(c-b)^2-2am(c-b)+a^2=m^2(c-b)^2+m^2(a^2-r^2)+(c-b)^2+a^2-r^2=0$ $-2am(c-b)=a^2m^2-m^2r^2+(c-b)^2-r^2=0$ $(1+m^2)r^2=(c-b)^2+2am(c-b)+a^2m^2$ $(1+m^2)r^2=(c-b+am)^2$ as required.	
16a	If $y=e^{\tan^{-1}2x}$ show that $(1+4x^2)\frac{d^2y}{dx^2}+2(4x-1)\frac{dy}{dx}=0$ $y=e^{\tan^{-1}2x}$, $\frac{dy}{dx}=\frac{2}{(1+4x^2)}.e^{\tan^{-1}2x}$ so $\frac{dy}{dx}=\frac{2y}{(1+4x^2)}$, $\frac{d^2y}{dx^2}=\frac{2(1+4x^2)\frac{dy}{dx}-2y.8x}{(1+4x^2)^2}$ $(1+4x^2)\frac{d^2y}{dx^2}=2\frac{dy}{dx}-8x.\left(\frac{2y}{(1+4x^2)}\right)$, $(1+4x^2)\frac{d^2y}{dx^2}=2\frac{dy}{dx}-8x.\left(\frac{dy}{dx}\right)$ $(1+4x^2)\frac{d^2y}{dx^2}+2(4x-1)\frac{dy}{dx}=0$	
b)	The displacement of a particle at time t is x, measured from a fixed point and $\frac{dx}{dt}=a(c^2-x^2)$, where a and c are positive constants, and $x=0$ when $t=0$. Prove that $x=\frac{c(e^{2act}-1)}{e^{2act}+1}$. If $x=3$ when $t=1$ and $x=\frac{75}{17}$ when $t=2$, prove that $c=5$	

	and find the value of a. $\frac{dx}{dt}=a(c^2-x^2), \quad \int \frac{dx}{(c+x)(c-x)}=\int a\,dt$ Let $1\equiv A(c-x)+B(c+x)$, thus $A=B=\frac{1}{2c}$ $\frac{1}{2c}\int \frac{1}{(c+x)}dx+\frac{1}{2c}\int \frac{1}{(c-x)}dx=\int a\,dt$ $\frac{1}{2c}In\frac{c+x}{c-x}=at+k$, for $t=0, x=0, k=0$ $\frac{1}{2c}In\frac{c+x}{c-x}=at$ $c+x=e^{2act}(c-x)$, thus $x=\frac{c(e^{2act}-1)}{e^{2act}+1}$ as required. $t=1, x=3$ $\frac{1}{2c}In\frac{c+3}{c-3}=a\ldots$(i) $t=2, x=\frac{75}{17}$ $\frac{1}{2c}In\frac{17c+75}{17c-75}=2a\ldots$(ii) $\frac{17c+75}{17c-75}=\frac{(c+3)^2}{(c-3)^2}$, $(17c+75)(c^2-6c+9)=(17c-75)(c^2+6c+9)$ $17c^3-27c^2-297c+675=17c^3+27c^2-297c-675$ $54c^2=1350$ thus $c^2=25$ so $c=\pm 5$ since it was positive then $c=5$	