<u>**TRANSFORMATIONS(TRANSLATION,REFLECTION& ROTATION)**</u>.

A transformation is an operation that changes the position of a point or shape. In some transformations the size of the shape is also altered.

Our discussion will be limited to only the following transformations: ***Translation, reflection, rotation and enlargement.***

1. <u>**TRANSLATION.**</u>

A translation is described by the movement of a point in the x and y directions as observed on the x – y plane.

Example 1 :

If a triangle P(0,0) Q (4,0)R (4,2) under goes a translation of $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$, find its image

Solution

The vector $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ describes a movement through + 3 units along the x –axis and then +5 units along the y-axis.

Thus the image points will be given by

P'; $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ <u>P'(3, 5)</u>

Q' ; $\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$ <u>Q'(7,5)</u>

R' ; $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix}$ <u>R'(7, 7)</u>

Example 2 : The image of triangle ABC after a translation T is A'(4, 6)B'(7, 8)C'(9,4)

Given that the object point A is the point(1, 3) find the coordinates of B and C.

Let the translation vector be $\begin{pmatrix} a \\ b \end{pmatrix}$

If $\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Then $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$B \begin{pmatrix} x \\ y \end{pmatrix} + T\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$

$B \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$; <u>B(4, 5)</u>

$C \begin{pmatrix} x \\ y \end{pmatrix} + T\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

$C \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$; <u>C(6,1)</u>

Example 3.

Find the image of the unit square O (0 ,0) B (1,0) C(1,1) D(0,1)

after a translation $T \begin{pmatrix} -5 \\ -7 \end{pmatrix}$

Followed by $S \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ followed by $R \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Solution

O' ; $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 \\ -7 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$; O' (-3, 0)

B ; $\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -5 \\ -7 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$; B' (-2, 0)

Note: the translations can be combined to give a single translation representing the net effect.

$$\begin{pmatrix}-5\\-7\end{pmatrix} + \begin{pmatrix}0\\4\end{pmatrix} + \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}-3\\0\end{pmatrix}$$

This is denoted as translation RST $= \begin{pmatrix}-3\\0\end{pmatrix}$

C' ; $\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} = \begin{pmatrix}-2\\1\end{pmatrix}$; C' (-2, 1)

D' ; $\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$; D' (-3, 1)

Note: If a translation T is repeated two times one after the other, then the combined effect is denoted as T^2.

Exercise*:(1). R, S, T are translations described as* $R = \begin{pmatrix}4\\3\end{pmatrix}$, $S = \begin{pmatrix}-2\\-2\end{pmatrix}$, $T = \begin{pmatrix}3\\5\end{pmatrix}$

Given that triangle ABC has vertices A(-1,1),B(-4,1),C(-1,3) and UVW has vertices U(2,2),V(6,2),W(6,4).

(a) Find the image of ABC under (i) RST (ii) S^2.

(b)Find the image of UVW under STR.

2. **REFLECTION**

When you look into a mirror you quickly notice a few things:

a) your image body is the same distance away from the mirror as you are,
b) the parts of your body are directly opposite to each other(i.e the image nose points directly to your nose and the line joining any two corresponding points is perpendicular to the reflecting surface of the mirror

Method of reflection:

a) Plot the object point(s) on the x – y plane
b) Construct the reflection (mirror) line on the same (x-y) plane and label this line (write its equation along the line).

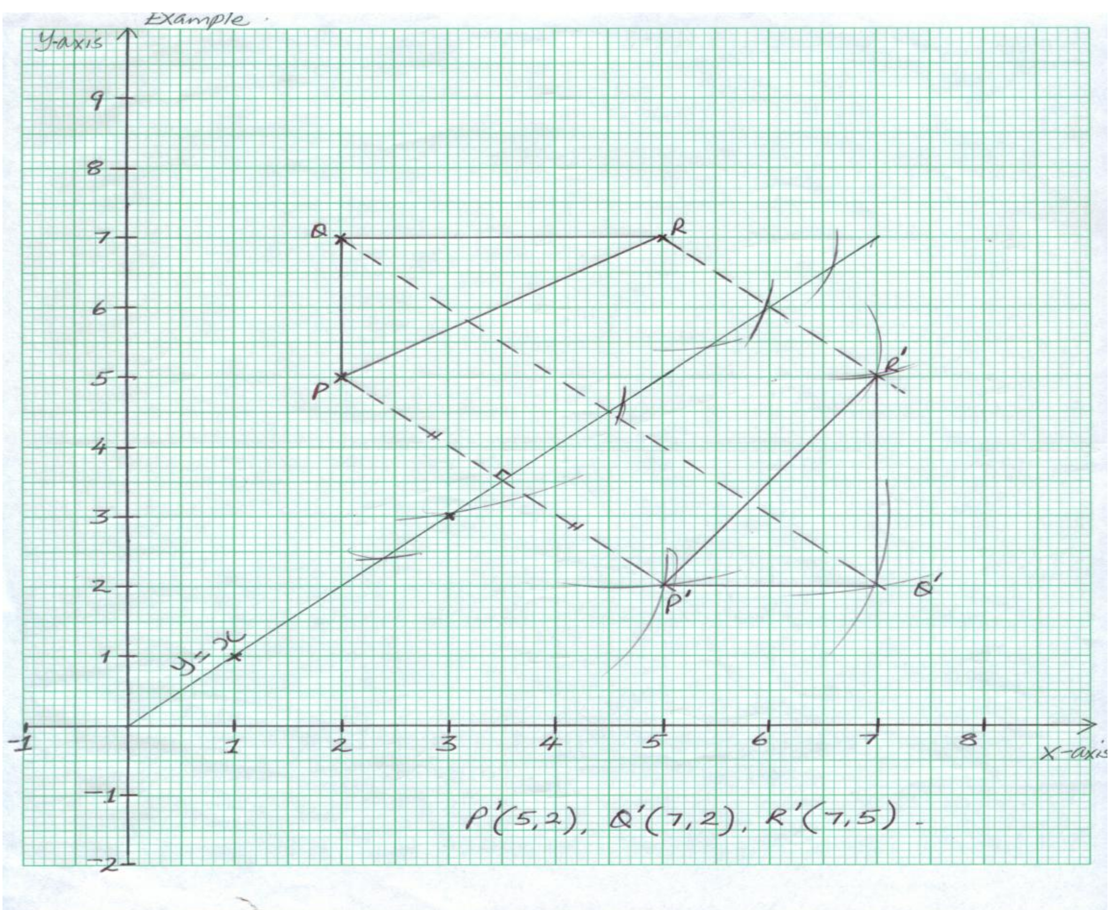
c) Pick your compass from the geometry set, position the compass at the object point and stretch the radius of the compass and cut the mirror line on two different points with visible crosses.

d) Without changing the radius, lift the compass and position it onto one of the crosses and make an arc on the other side of the mirror line opposite to the object. Then transfer the compass on the second cross made on the mirror line and still with the same radius, make a second arc and let it intersect with the first arc. The intersection of the arcs will be the image, I, of the object, O.

Example 4. Reflect the triangle P(2,5),Q(2,7),R(5,7) through the line y = x.

The line y = x can be drawn using the points in the table below.

x	0	1	3
y	0	1	3



Exercise *:(2) Try to follow the steps in the method above)*

Find the image of R (-2, 4) S (1,-3) T (3, 2) after a reflection in the line $y = 2x + 1$

Note: When drawing the line, construct a table of values with at most three points.

e.g

x	0	1	-2
y	1	3	-3

Example 5. ***Finding the mirror line***.

Given that when the triangle A(2, 5) B(7, 2) C(5, 7) is reflected through a given line its image is A'(5,2) B'(2,7) C'(7,5).

Find the mirror line and its equation

Method :- Plot both the object and image on the same x – y plane.

a) Join one of the objects to its image (A to A')
b) Bisect the line joining the object to the image

This bisector will be the mirror line.

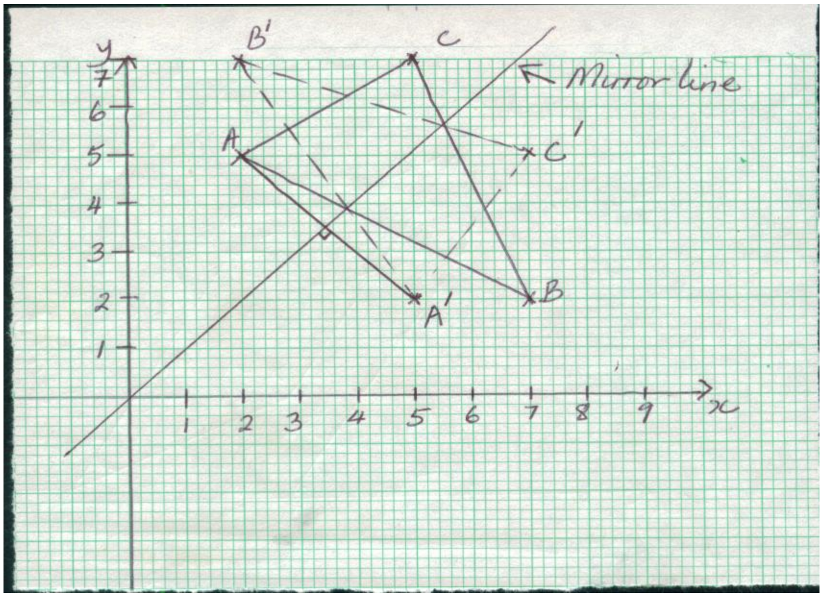
c) (i)List down the coordinates of the mid point of the line joining the object to the image, (AA') or any convenient point on the line(Let this be the point (x_1,y_1)).

   (ii)Find the coordinates of the point at which the bisector cuts the y – axis, this is of the form (0,c). Let this be the point(x_2, y_2).

d) Calculate the gradient of this line, m, from:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

e) Use the general equation of a line $y = mx + c$



From above using (x_1, y_1) as (3, 3) and (x_2,y_2) as (0, 0)

$$m = \frac{0 - 3}{0 - 3} = \frac{-3}{-3} = 1$$

Using y = mx + c

:. y = 1(x) + 0

<u>y = x</u> (The equation of the mirror line)

<u>Summary of common reflections</u>

Mirror line	**Nature of image as compared to object**
a) x – axis	- The x remains the same but y changes sign (2, -3) → (2, 3)
d) y – axis	- The y remains the same but x changes sign (4, 5) →(- 4, 5)
c) x = y	- The x becomes the new y and the y becomes the new x (7, 3) → (3, 7)
d) x = -y or x + y = 0	- The negative of x becomes the new y and the negative of y becomes the new x. (6,9) → (-9,-6) Note. This summary should be used to help you check your answers and not as a substitute to graphical work.

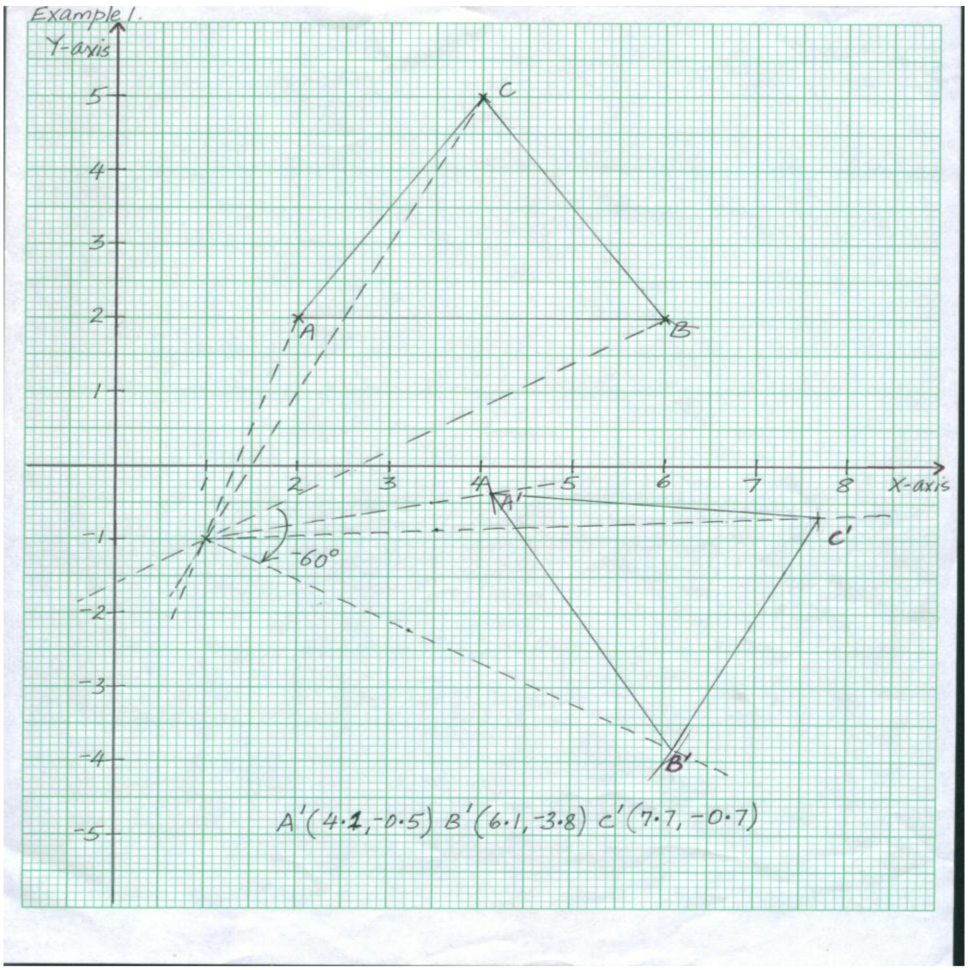
Exercise: (3) R is a reflection in the line x + y = 0.*T is a translation which maps (-1,-1) onto (2,0).Find the image of A(0,2),B(2,2),C(0,4) under (i)*RT^2 *(ii)* T^3.

3. ROTATION.

A rotation is a turn described by the angle turned through (angle of rotation) and the point at which the turning is done (Centre of rotation). The angle of rotation may be positive (turn in the anticlockwise sense) or negative (turn in the clockwise sense)

Example 1.

Find the image of A (2, 2), B(6,2), C(4, 5) after a rotation through -60^0 about the point (1, -1)



Method of rotation **:**

a) plot the object points and the centre of rotation on the same x – y plane.

b) Join A to the centre of rotation and extend the line slightly. Measure the distance of A from the centre of rotation

c) Place your protractor at the centre of rotation along this line running from A to the centre so as to measure the required angle (take note of the positioning of the protractor to be able to measure positive and negative angles)

d) Measure the angle and remove the protractor. Trace out the line showing the measured angle. (This is the image line).

e) Measure off the same distance as that of the object from the centre, along the image line. (Using a compass, place it at the centre and stretch it's radius up to the object. Turn the compass without lifting it off the centre and cut off the same radius along the image line).This will be the image point.

f) Repeat (b) to (e) for all the other points.

Example 2. ***<u>Finding the centre and angle of rotation.</u>***

The image of the square A(2,1) B(4,1) C(4,3) D(2,3), after a rotation is A' (-2,-2) B'(-2,1) C'(-4,1) D' (-4,-1).Find the centre and angle of rotation. (Solution on the graph below)

Method:

a) Plot both the object and image on the same x-y plane.

b) Join A to A' to form the line, AA'. Repeat for any other point say, BB'.

c) Bisect these two lines formed i.e. AA' and BB'.

d) The point of intersection of the bisectors is the centre of rotation.

e) To measure the angle; join one of the points to the centre (object line) and its image to the centre (image line)

With an arrow show the "turn" from the object line to the image line (arrow shows whether the turning is anticlockwise or clockwise). Then measure the magnitude of this angle.

Exercise*:(1) Draw triangle ABC at A(2,4),B(6,10),C(12,4).Find the image of ABC after a rotation through* ***-45^0*** *about (0,6).*

(2) Draw the object triangle A(3,1),B(6,1),C(6,3).Describe fully the rotation which maps ABC onto A'(4,4),B'(1,4),C'(1,2).