# UNDERSTANDING CIRCLE THEOREMS-PART ONE.

Common terms:

(a) **ARC**- Any portion of a circumference of a circle.

(b) **CHORD-** A line that crosses a circle from one point to another. If this chord passes through the centre then it is referred to as a diameter

(c) **A TANGENT**- A line that touches a circle at only one point.

Theorem 1.

**The angle subtended at the centre of a circle is twice the angle subtended at the circumference by the same arc.**

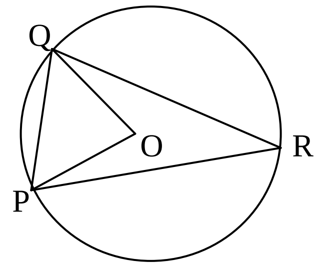
Theorem 2.

**Angles subtended by an arc in the same segment of a circle are equal.**

*Example 1.*

Given PQO = $65^0$

Find QRP



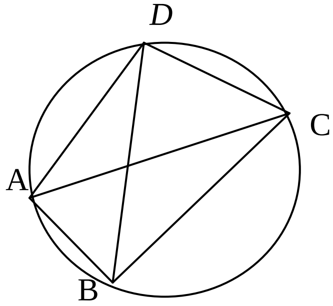

Triangle OQP is isosceles (OP = OQ, the radii)

:. OPQ = $65^0$ :. QOP = $180^0 - (65^0 + 65^0)$ (angle sum of a triangle)

= $50^0$

:. QRP = $25^0$ (half of angle at the centre).

*Example2.*




Given that angle BDC = $78^0$ and DCA = $56^0$.

Find angles BAC and DBA.

**Solution**: BAC = BDC = $78^0$. ( both subtended by arc BC)

DBA = DCA = $56^0$. ( both subtended by arc AD)

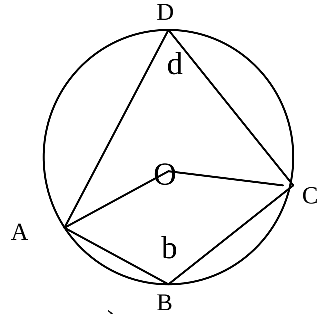
<u>Theorem3</u>.

**The opposite angles in a cyclic quadrilateral add up to $180^0$ (the angles are supplementary).**

ABCD is a **cyclic quadrilateral** because all its vertices touch the circumference of the circle.(ABCO is not cyclic because O is not at the circumference).

<u>Proof:</u>

OA and OC are radii.
Let angle ADC = d
and angle ABC = b




AOC obtuse = 2d (angles at the centre)

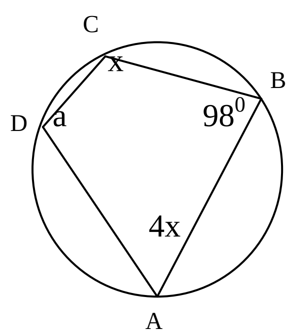
AOC reflex = 2b (angles at the centre)

:. 2d + 2b = $360^0$ (angles at a point)

:. d + b = $180^0$ as required

*Example3.*

Find *a* and *x*




CDA = a, ABC = $98^0$ , DCB = $x^0$ ,DAB = $4x^0$

a = $180^0 - 98^0$ (opposite angles of a cyclic quadrilateral)

:. a = $82^0$

x + 4x = $180^0$ ( opposite angles of a cyclic quadrilateral

5x = $180^0$

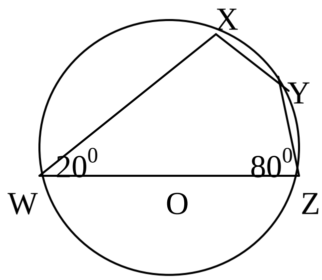
x = $72^0$

***Exercise***. 1.ABCD is a quadrilateral inscribed in circle, centre O, and AD is a diameter of the circle. If angle CDB = $46^0$ and ADB = $31^0$. Calculate

(a) the angle ABC (b) the angle BCD (c) the angle BAD.

2. A circle has a radius of 155mm .AB is a chord of this circle which is 275mm long. What angle does AB subtend at the circumference of the circle.

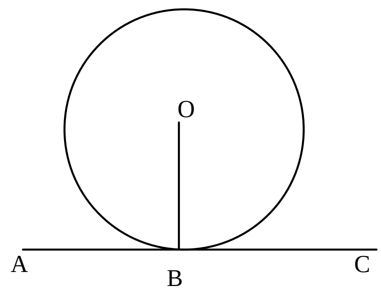
3. Given angle XWZ = $20^0$ , angle WZY = $80^0$ and O is the centre of the circle
   (a) Find angle WXY
   (b) Show that WY bisects XWZ



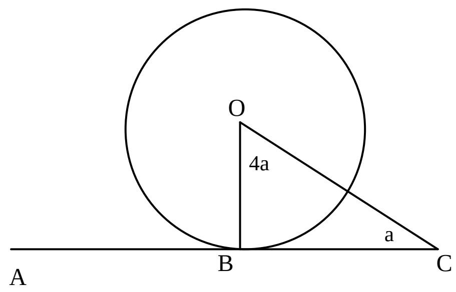

<u>Theorem 4.</u>

**The angle between a tangent and the radius drawn to the point of contact is $90^0$**

Line ABC is a tangent and angle ABO = $90^0$




*Example 4.* Find the angle BCO and angle BOC.




$$4a + a + 90^0 = 180^0$$

$$5a = 90^0$$

$$a = 18^0.$$

Angle BCO = $18^0$ and BOC = $4 \times 18^0 = 72^0$.

# UNDERSTANDING CIRCLE THEOREMS –PART TWO.

Theorem 5.

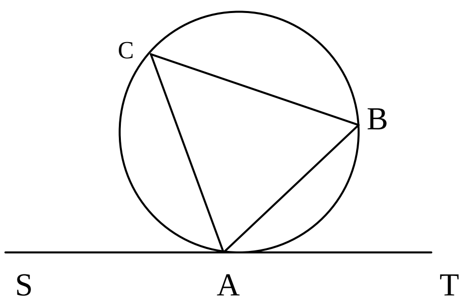
**The tangents to a circle originating from a common point are equal in length.**

Theorem 6.

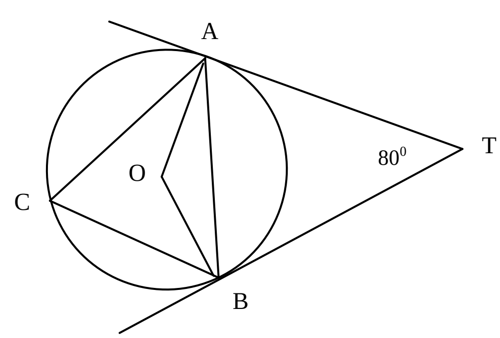
The Alternate segment theorem.

**The angle between a tangent and chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.**

angle TAB = angle BCA and angle SAC = angle CBA




*Example 1.*




a) △ TBA is isosceles (TA = TB) ,angle TAB = angle TBA.

:. TBA = ½ (180 – 80)

= $50^0$

b) $OBT = 90^0$ (tangent and radius)

$OBA = 90^0 - 50^0$

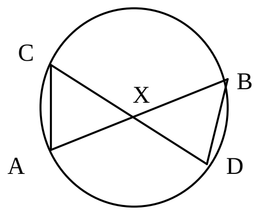
$= 40^0$.

c) ACB = ABT (alternate segment theorem)

$ACB = 50^0$

Theorem 7.

**Intersecting chords theorem**




**AX.BX = CX.DX**

**Proof:**

In triangles AXC and BXD:

Angle ACX = angle DBX (same segment)

Angle CAX = angle BDX (same segment)

:. The triangles AXC and BXD are similar.

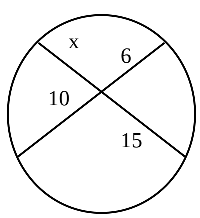
$$\frac{AX}{DX} = \frac{CX}{BX}$$

Thus **AX.BX = CX.DX**

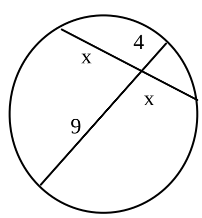
*Exercise.*

Find x

(a)




(b)

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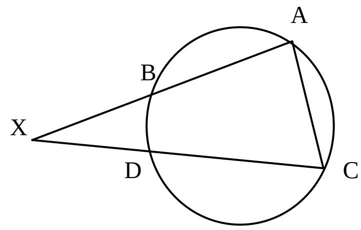
Theorem 8. **The intersecting secants theorem.**




Using triangles BXD and AXC;

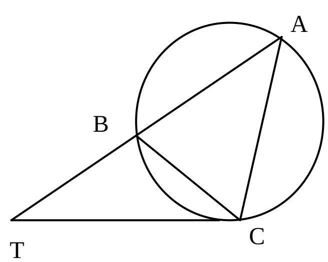
Angle XAC = angle XDB, angle XCA =angle XBD.(Figure BACD is a cyclic quadrilateral).Thus triangles AXC and BXD have equal angles and are similar.

$$\frac{AX}{DX} = \frac{CX}{BX}$$

Thus **AX.BX = CX.DX (This is the intersecting secants theorem)**

Theorem 9.

**The secant/tangent theorem.**

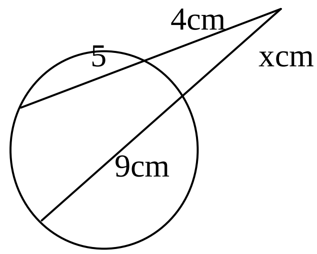



Angle BCT = angle BAC(alternate segment theorem). Triangles ATC and BTC share angle T and are similar triangles. (when triangles have two angles equal then they are similar.

In the triangles,

$\frac{AT}{TC} = \frac{TC}{BT}$ ;**AT.BT = $TC^2$(This is the secant/tangent theorem).**

*Example 2*.




Solution: $4 \times 9 = x.(9 + x)$

$36 = 9x + x^2.$

$x^2 + 9x - 36 = 0$

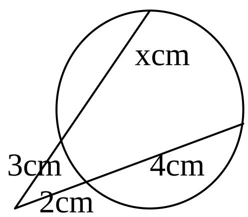
$x^2 + 12x - 3x - 36 = 0$

$x(x + 12) - 3(x + 12) = 0$

$(x - 3)(x + 12) = 0$; $x = 3$ or $x = {}^-12$.

Since x cannot be negative then <u>x = 3 cm.</u>

*Example 3*.




Solution: $3(3 + x) = 2 \times 6.$

$9 + 3x = 12.$

$3x = 3$

<u>x = 1cm</u>.

Exercise.

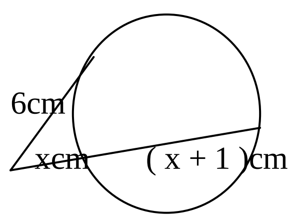
1. Two chords of a circle KL and MN intersect at X, and KL is produced to T. Given that KX = 6cm, XL = 4cm, MX = 8cm and LT = 8cm. calculate

   a) NX

   b) The length of the tangent from T to the circle

   c) The ratio of the areas of $\triangle$ KXM to $\triangle$ LXN.

2.




Find x.

3. Chords AB and BC of a circle are produced to meet outside the circle at T. a tangent is drawn from T to touch the circle at E. Given AB = 5cm, BT = 4cm and DC = 9cm, Calculate

a) CT b) TE

c) the ratio of the areas of $\triangle$ADT to $\triangle$BCT

d) the ratio of the areas of $\triangle$ BET to $\triangle$ AET