

MATHEMATICS HOLIDAY WORK

FOR S.1, S.2, S.3 AND S.4

WED, 08TH JAN 2014.

INSTRUCTIONS:

1. Read the notes below on fractions carefully and answer the exercise on pages 11 and 12.
2. Note the mistakes pointed out and avoid repeating them in the exercise.
3. Put your answers on separate answer sheets and come with them on the opening day of the term.
4. Show your working clearly and neatly.
5. S.3M and S.4T students, scan your work and send it to my e-mail “kennedymatumbwe@yahoo.com” before 28th Jan 2014.

This work on fractions is part of what I have put in my book **‘Common mistakes in mathematics’** which will be out in **February this year(2014)**. This book shows most of the mistakes different students make in mathematics.

HAPPY NEW YEAR TO EVERY ONE

FRACTIONS

A fraction is part of a whole number.

There are three major types of fractions namely;

- (1) **Proper fraction:** This is where the numerator is smaller than the denominator. For example, $\frac{1}{2}, \frac{8}{9}, \frac{21}{37}, \frac{99}{100}$ etc.
- (2) **Improper fraction:** This is where the numerator is greater than the denominator. For example; $\frac{3}{2}, \frac{11}{9}, \frac{79}{23}, \frac{135}{2}$ etc.
- (3) **Mixed fraction:** This contains a whole number and a fraction part. For example; $3\frac{2}{5}, 34\frac{1}{2}, 111\frac{7}{8}$ etc.

OPERATIONS WITH FRACTIONS

(a) Addition and subtraction.

Examples

Work out the following;

(i) $\frac{2}{5} + \frac{1}{5}$ (ii) $\frac{3}{4} + \frac{2}{3}$ (iii) $\frac{5}{7} - \frac{1}{4}$

(iv) $5\frac{3}{4} - 3\frac{2}{3}$ (v) $\frac{3}{5} - \frac{7}{8} + \frac{1}{2}$

Answers

(i) $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5}$ (Since the denominator is the same, we only add the numerators)
 $= \frac{3}{5}$

(ii) $\frac{3}{4} + \frac{2}{3} = \frac{9+8}{12}$ (LCM of 4 and 3 is 12)
 $= \frac{17}{12}$
 $= 1\frac{5}{12}$ (Always change your final answer from improper to mixed fraction)

$$(iii) \quad \frac{5}{7} - \frac{1}{4} = \frac{20-7}{28} \quad (\text{LCM of 7 and 4 is 28})$$

$$= \frac{13}{28} \quad (\text{The answer is a fraction but not } \frac{13}{28} \text{ as some students carelessly write it})$$

$$(iv) \quad 5\frac{3}{4} - 3\frac{2}{3} = (5-3) + \left(\frac{3}{4} - \frac{2}{3}\right) \quad (\text{one can first handle whole numbers and fractions separately})$$

$$= 2 + \frac{9-8}{12} \quad (\text{LCM of 4 and 3 is 12})$$

$$= 2 + \frac{1}{12}$$

$$= 2\frac{1}{12}$$

Method 2

$$5\frac{3}{4} - 3\frac{2}{3} = \frac{(5 \times 4) + 3}{4} - \frac{(3 \times 3) + 2}{3} \quad (\text{changing mixed to improper fractions})$$

$$= \frac{23}{4} - \frac{11}{3}$$

$$= \frac{69-44}{12} \quad (\text{LCM of 4 and 3 is 12})$$

$$= \frac{25}{12}$$

$$= 2\frac{1}{12} \quad (\text{Final answer in mixed fraction})$$

$$(v) \quad \frac{3}{5} - \frac{7}{8} + \frac{1}{2}$$

Many students make a mistake on such numbers; see how they work out this number;

$$\frac{3}{5} - \frac{7}{8} + \frac{1}{2} = \frac{3}{5} - \frac{7+4}{8} \quad \text{BODMAS}$$

$$= \frac{3}{5} - \frac{11}{8}$$

$$= \frac{24-55}{40} \quad (\text{LCM of 8 and 5 is 40})$$

$$= -\frac{31}{40} \quad \left(\text{This is a wrong answer because you have considered } \frac{1}{2} \text{ to be negative}\right)$$

by adding it $-\frac{7}{8}$ and you take the sum to be negative)

See the correct working below:

$$\begin{aligned} \frac{3}{5} - \frac{7}{8} + \frac{1}{2} &= \frac{3}{5} + \frac{1}{2} - \frac{7}{8} \\ &= \frac{6+5}{10} - \frac{7}{8} && \text{BODMAS} \\ &= \frac{11}{10} - \frac{7}{8} \\ &= \frac{44-35}{40} && \text{(LCM of 10 and 8 is 40)} \\ &= \frac{9}{40} \end{aligned}$$

(b) Multiplication and Division

Examples

Work out the following:

(i) $\frac{3}{5} \times \frac{5}{6}$ (ii) $3\frac{1}{2} \times 4\frac{1}{4} \times 1\frac{5}{6}$ (iii) $\frac{4}{9} \div \frac{2}{5}$

(iv) $2\left(\frac{3}{5}\right)$

Answers

$$\begin{aligned} \text{(i)} \quad \frac{3}{5} \times \frac{5}{6} &= \frac{3 \times 5}{5 \times 6} \\ &= \frac{15}{30} \\ &= \frac{1}{2} && \text{(Reduced to the simplest term)} \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad 3\frac{1}{2} \times 4\frac{1}{4} \times 1\frac{5}{6} &= \frac{(3 \times 2) + 1}{2} \times \frac{(4 \times 4) + 1}{4} \times \frac{(1 \times 6) + 5}{6} \\
 &= \frac{7}{2} \times \frac{17}{4} \times \frac{11}{6} \quad \text{(Changed to improper fractions)} \\
 &= \frac{7 \times 17 \times 11}{2 \times 4 \times 6} \\
 &= \frac{1309}{48} \\
 &= 27\frac{13}{48}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{4}{9} \div \frac{2}{5} &= \frac{4}{9} \times \frac{5}{2} \quad \text{(Introduce a multiplication sign and the reciprocal of the next} \\
 &\hspace{15em} \text{Fraction)} \\
 &= \frac{4 \times 5}{9 \times 2} \\
 &= \frac{20}{18} \\
 &= 1\frac{1}{9}
 \end{aligned}$$

$$\text{(iv)} \quad 2\left(\frac{3}{5}\right)$$

Some students confuse multiplying a column vector by a scalar with multiplying a fraction by a whole number. See how they carelessly answer this;

$$\begin{aligned}
 2\left(\frac{3}{5}\right) &= \frac{2 \times 3}{2 \times 5} \\
 &= \frac{6}{10} \\
 &= \frac{3}{5} \quad \text{Wrong answer.}
 \end{aligned}$$

Follow the correct working below;

$$\begin{aligned}
 2\left(\frac{3}{5}\right) &= \frac{2 \times 3}{5} \\
 &= \frac{6}{5}
 \end{aligned}$$

$$= 1\frac{1}{5}$$

(c) Combined operations

The order of operations used for fractions is the same as that used for whole numbers, i.e work out the brackets first, then of, divide, multiply, add and lastly subtract (BODMAS).

Examples

Evaluate the following:

$$(i) \quad \frac{5}{8} \text{ of } \frac{4}{15} \div \frac{7}{10} + \frac{2}{5}$$

$$(ii) \quad \frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}}$$

Answers

$$\begin{aligned} (i) \quad \frac{5}{8} \text{ of } \frac{4}{15} \div \frac{7}{10} + \frac{2}{5} &= \frac{20}{120} \div \frac{7}{10} + \frac{2}{5} && \text{(BODMAS)} \\ &= \frac{1}{6} \div \frac{7}{10} + \frac{2}{5} \\ &= \frac{1}{6} \times \frac{10}{7} + \frac{2}{5} \\ &= \frac{5}{21} + \frac{2}{5} \\ &= \frac{25 + 42}{105} \\ &= \frac{67}{105} \end{aligned}$$

$$(ii) \quad \frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}}$$

Many students find it difficult to work out such expressions. See below the mistake they commonly make;

$$\frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}} = 2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2} \div 4\frac{1}{6} - 2\frac{1}{5} \quad \left(\text{This is wrong unless if } 2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2} \text{ and} \right.$$

$4\frac{1}{6} - 2\frac{1}{5}$ are put in separate brackets)

$$= \frac{5}{2} + \frac{13}{3} - \frac{5}{2} \div \frac{25}{6} - \frac{11}{5}$$

$$\begin{aligned}
&= \frac{5}{2} + \frac{13}{3} - \frac{5}{2} \times \frac{6}{25} - \frac{11}{5} \quad (\text{BODMAS}) \\
&= \frac{5}{2} + \frac{13}{3} - \frac{3}{5} - \frac{11}{5} \\
&= \frac{75 + 130 - 18 - 66}{30} \\
&= \frac{121}{30} \\
&= 4\frac{1}{30} \quad (\text{Wrong})
\end{aligned}$$

See the different correct ways of evaluating this expression;

(i)

$$\begin{aligned}
\frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}} &= \frac{\frac{5}{2} + \frac{13}{3} - \frac{5}{2}}{\frac{25}{6} - \frac{11}{5}} \\
&= \frac{15 + 26 - 15}{125 - 66} \\
&= \frac{26}{59} \\
&= \frac{26}{6} \div \frac{59}{30} \\
&= \frac{26}{6} \times \frac{30}{59} \\
&= \frac{130}{59} \\
&= 2\frac{12}{59}
\end{aligned}$$

(II)

$$\frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}}$$

(First simplify the numerator and denominator separately and later combine them again)

$$\begin{aligned}\text{Numerator} &= 2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2} \\ &= \frac{5}{2} + \frac{13}{3} - \frac{5}{2} \\ &= \frac{15 + 26 - 15}{6} \\ &= \frac{26}{6} \\ &= \frac{13}{3}\end{aligned}$$

$$\begin{aligned}\text{Denominator} &= 4\frac{1}{6} - 2\frac{1}{5} \\ &= \frac{25}{6} - \frac{11}{5} \\ &= \frac{125 - 66}{30} \\ &= \frac{59}{30}\end{aligned}$$

$$\begin{aligned}\text{Combining;} \quad \frac{2\frac{1}{2} + 4\frac{1}{3} - 2\frac{1}{2}}{4\frac{1}{6} - 2\frac{1}{5}} &= \frac{13}{3} \div \frac{59}{30} \\ &= \frac{13}{3} \times \frac{30}{59} \\ &= \frac{130}{59} \\ &= 2\frac{12}{59}\end{aligned}$$

RECURRENCE DECIMALS

When fractions are changed to decimals, some of them give a definite number of decimal places. For example; $\frac{1}{2} = 0.5$, $\frac{1}{4} = 0.25$, $\frac{3}{8} = 0.375$ etc. Those are called terminating decimals.

However, there are some fractions that give decimals that never end. For example; $\frac{1}{3} = 0.3333\dots$, $\frac{8}{33} = 242424\dots$ etc.

When we change $\frac{1}{3}$ to a decimal, the digit 3 in the decimal repeats itself continuously without ending. We say that 3 is a recurring digit. Decimals with such a recurring digit are called recurring decimals.

Recurring decimals can be written in the following ways;

- (i) $0.333\dots$ or $0.\dot{3}$ or $0.\bar{3}$
(ii) $0.2424\dots$ or $0.\dot{24}$ or $0.\overline{24}$
(iii) $3.717171\dots$ or $3.\dot{71}$ or $3.\overline{71}$

Examples

Express the following recurring decimals as fractions in their simplest terms.

- (1) $0.\dot{63}$ (2) $0.\overline{321}$

Answers:

- (1) $0.\dot{63}$

Many students forget to put symbols to show that the decimal is a recurring decimal and this makes their working wrong since for

example $0.63 \neq 0.\dot{63}$ and $0.63 = \frac{63}{100}$

See both workings below:

Wrong

$$\begin{aligned} \text{Let } x &= 0.63 \\ 100 \times x &= 0.63 \times 100 \\ 100x &= 63.63 \\ 100x - x &= \begin{array}{r} 63.63 \\ -0.63 \end{array} \\ 99x &= 63 \\ x &= \frac{63}{99} \\ x &= \frac{7}{11} \end{aligned}$$

This is wrong b'c the recurring symbol is not seen anywhere. The recurring symbol must appear at least once in your working.

Correct

$$\begin{aligned} \text{Let } x &= 0.\overline{63} \\ 100 \times x &= 0.6363\dots \times 100 \\ 100x &= 63.6363\dots \\ 100x - x &= \begin{array}{r} 63.6363\dots \\ -0.6363\dots \end{array} \\ 99x &= 63.0000 \\ x &= \frac{63}{99} \\ x &= \frac{7}{11} \end{aligned}$$

(2) $0.\overline{321}$

Let $r = 0.321321\dots$

Multiply both sides by 1000 because of three recurring digits in the decimal given.

$$\begin{aligned} 1000 \times r &= 0.321321\dots \times 1000 \\ 1000r &= 321.321321\dots \\ 1000r - r &= \begin{array}{r} 321.321321\dots \\ -0.321321\dots \end{array} \\ 999r &= 321.000000 \\ r &= \frac{321}{999} \\ r &= \frac{107}{333} \end{aligned}$$

EXERCISE ON FRACTIONS:

(1) Simplify the following:

(a) $\frac{3}{4} + \frac{1}{5}$ (b) $\frac{2}{x} + \frac{3}{2x}$ (c) $2\frac{2}{3} + 1\frac{3}{5}$ (d) $4\frac{1}{2} - 3\frac{1}{9}$

(e) $\frac{7\frac{1}{5} \times 5\frac{1}{7}}{7\frac{1}{5} - 5\frac{1}{7}}$ (f) $2\frac{1}{2} \div \frac{4\frac{1}{3} - 2\frac{1}{4}}{4\frac{1}{6}}$ (g) $\frac{(3\frac{1}{7} \times 2\frac{1}{3}) - 2}{4\frac{1}{2} - 3\frac{1}{9}}$

(h) $\frac{2x-3}{2} - \frac{x+4}{5} - \frac{x-5}{4}$ (i) $\frac{1-x}{x} - \frac{1-y}{y}$

(j) $\frac{2\frac{1}{3} - 1\frac{1}{5}}{5\frac{1}{5} + \frac{3}{4}}$

(2) Find the value of $\left(4\frac{1}{2} + \frac{2}{3}\right) - \left(4\frac{1}{2} \times \frac{2}{3}\right)$.

(3) Evaluate $\frac{\frac{3}{5} \text{ of } 30 + 5\frac{5}{6} \div \frac{7}{12} - 2\frac{2}{3} \times 1\frac{1}{2}}{5\frac{5}{8} \times 1\frac{7}{9} - \frac{5}{9} \text{ of } 4\frac{4}{5} + \frac{14}{5} \div \frac{7}{10}}$. Leaving your answer in fraction form.

(4) . Express 0.341666.....as a fraction, in the form, $\frac{p}{q}$, where $q \neq 0$.

(5) . Simplify: $\frac{x}{x^2+3x} + \frac{x}{x^2+9x+18}$.

(6) Simplify: $3\frac{2}{3} + 1\frac{1}{2} - 4\frac{1}{4}$.

- (7) Express 0.1818.... as a common fraction.
- (8) Given that $a = 1\frac{1}{2}$, $b = \frac{3}{4}$, and $c = 2\frac{1}{2}$, evaluate: $c + a \div b$.
- (9) Joseph sweeps $\frac{1}{4}$ of a compound, Milly sweeps $\frac{3}{5}$ of the same compound and Susan sweeps the rest.
- (a) What fraction does Susan sweep?
 (b) If Susan sweeps 21 square metres, how big is this compound?
10. **Evaluate** $\frac{3}{8}$ of $\left\{7\frac{3}{5} - \frac{1}{3}\left(1\frac{1}{4} + 3\frac{1}{3}\right) \times 2\frac{2}{5}\right\}$
11. **Express** $4.15\dot{2}4\dot{3}$ as a fraction in its simplest form.
12. Two boys and a girl shared some money. The younger boy got $\frac{5}{18}$ of it, the elder boy got $\frac{7}{12}$ of the remainder and the girl got the rest. **Find** the percentage share of the younger boy to the girl's share.
13. Evaluate $\left(\frac{2\frac{1}{4} - 3\frac{3}{4} \times 3\frac{2}{3} \div 2\frac{1}{5}}{1\frac{5}{6} \div 1\frac{1}{4}}\right)$.
14. Convert $2.\dot{3}\dot{2}$ into a fraction giving your answer as a mixed fraction.
15. Ben spends $\frac{2}{3}$ of his income on food and $\frac{1}{5}$ of the remainder on rent and saves the rest. If he saves shs. 44,800, find Ben's income.

END