1. The diameter of an egg produced by a hen of certain farm depends on the mass of the layers mass ratio it is fed as shown in the table below;

| Food ratio(g) | 200 | 290 | 330 | 410 | 440 | 500 |
|---|---|---|---|---|---|---|
| Diameter(mm) | 30.2 | 34.2 | 36.2 | 40.1 | 41.0 | 46.2 |

Assuming the egg to be spherical, find the;

(i) optimum amount of the food the hen should be given if it is to produce an egg of average diameter of 37.5mm

(ii) radius of an egg if the food ratio supplied is 520g

2. A car moving with a speed of 16km/h in the direction -4i +3j, accelerates uniformly for 20 minutes. If the car was at (0, 1.2) km relative to the origin, find the;

    (i)   acceleration of the car

    (ii)   position of the car relative to the origin at the end of the 20 minutes.

3. A maize mill in Gayaza sells maize flour in bags of mean weight 50kg and standard deviation 2.5kg. Given that the weights of the bags are normally distributed, find the;

    (i)  probability that the weight of any bag of maize flour randomly selected lies between 51.5 and 53 kg

    (ii)  percentage of bags whose weight exceed 54kg

    (iii) number of bags that will be rejected out of 1000 bags purchased for weighing below 45kg.

4. A mass of 10kg rests on a smooth horizontal table. It is connected to two other masses of 8kg and 6kg by a light inextensible string passing over two smooth pegs fixed at opposite edges of the table. Initially the 8kg mass is 0.8m below the level of the table and the 6kg is on the ground. Given that the system is released from the rest, determine the;

a) common acceleration

b) velocity with which the 8kg hits the ground, if the ground is 2m below the surface of the table.

5. The continuous random variable X has a p.d.f f(x) where;

$$f(x) = \begin{cases} x/3 - 2/3 & 2 \le x \le 3 \\ \alpha & 3 \le x \le 5 \\ 2 - \beta x & 5 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Find (a)   the value of $\alpha$ and $\beta$

(b)   (i) F(x), the cumulative function of x

(ii)   Sketch y = F(x)

(c)   Find $P(2 \le x \le 3.5)$

6. A light elastic string has natural length 2m and modulus of elasticity 200N. The ends of the strings are attached to two fixed points P and Q which are on the same horizontal level 3m apart. An object is attached to the midpoint of the string and hangs in equilibrium at a point 0.5m below PQ. Calculate to 2 significant figures the;

a) mass of the object.

b) elastic potential energy stored in the string in this position.

7. A ship A is travelling on a course of 060° at a speed of $30\sqrt{3}$ km/h and a ship B is travelling on a course of 030° at 20km/h. At noon B is 260km due East of A. Find the time when A and B are closest together.

8. A lion sees a deer 100m due North running at a constant speed of 18km/h due S120°E.  If the lion can run at 36km/h, find the;

(i)   course the lion should take in order to intercept the deer

(ii)   time taken for the lion to catch the deer

9. In an examination 30% of the candidates fail and 10% achieve distinction. Last year the pass mark (out of 200) was 84 and the minimum mark required for a distinction was 154. Assuming that the mean marks of the candidates were normally distributed estimate the;
    (a) mean mark and the standard deviation.
    (b) Probability that a students scored between 120 and 175 marks

10. A random variable, R takes the integral value r with probability P(r),
    Where $P(r) = \begin{cases} Kr^3, & r = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$

    Find the; (a) value of K
            (b) mean and variance of R
            (c) mean and variance of $(5R - 3)$

11. A soft-drink machine is regulated so that it discharges an average of 200ml per cup. If the amount of drink is normally distributed with standard deviation 15ml, determine the;
    (i) fraction of the cups which contain more than 224ml
    (ii) probability that a cup contains between 191 and 209ml
    (iii) amount in cups below which we get 25% of the drinks

12. Box P contains 5 balls, 1 red, 3 green and 1 blue. Box Q contains 5 balls, 2 red, 1 green and 2 blue. A fair die is thrown and if the throw is a six, box P is chosen, otherwise Q is chosen. A ball is selected from the chosen box.
    (i) Find the probability that this ball is red
    (ii) Given that the ball is green, find the probability that it came from P.

13. (a) Derive the iterative formula based on Newton Raphson formula for finding the only root of the equation $2sinx = 2x - 1$

    (b) Construct a flowchart to illustrate the use of your formula in (a) above for;
    (i) reading the initial approximation $\alpha$ to the root
    (ii) computing and printing the root of the equation correct to 3 dps and after 3 iterations. Hence perform a dry run for your flow chart using $X_o = 1.2$

14. (a) A particle of mass 2kg is suspended by an inelastic thread of modulus 15N and natural length 15cm. If the particle is projected vertically downward, find its acceleration when the thread is stretched to a total length of 25cm.

(b) The same particle in (a) above is projected at 2m/s down a rough plane inclined at 30° to the horizontal. If the coefficient of friction between the particle and the plane is $1/\sqrt{3}$, find the time taken to slide 8m.

15. (a) (i) Truncate 180.00785 to 5 sfs

(ii) Round off 0.0000985 to 5 sfs

(b) (i) Find an expression for the relative error in $x\tan y$

(ii) If x = 2.5 and y=30°, find the range of values enclosing $x\tan y$ together with their error bounds.

(c) Determine the limits within which the exact value of $4^{\cos x}$ lies, where x=45°

16. Observation of a large number of buses at a certain point on a motor way indicates that the speeds are normally distributed. 90% of the buses have speeds of less 77.7 km/h and only 5% of buses have speeds less than 63.1 km/h.

(i) Find the mean and standard deviation of the buses speed, correct to 1 decimal place.

(ii) If 25 buses are selected at random, find the probability that their average speed is more than 72.5 km/h

(iii) If a sample of 10 buses is chosen, determine the probability that exactly three have speed between 75km/h and 80km/h

17. (a) Use the trapezium rule with six ordinates to estimate;

$$\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4\right) dx,$$ correct to 3 significant figures.

(b) Find the percentage error made in your estimation, giving your answer to 2 decimal places. Suggest how this error may be reduced.

18. A car of mass 1000kg travels at a constant speed of 60km/hr on a level road when the engine works at 15kW
     (i)   Determine the resistance to motion
     (ii)  With the same working and against the same resistance, the car now climbs a hill inclined at $\alpha$ to the horizontal at a constant speed of 45km/hr. Find the value of $\alpha$.

19. The table shows the costs to the nearest pound (£) of 400 text books which were bought by a certain university.

| Cost, C (£) | No. of Books |
|---|---|
| C < 5 | 11 |
| 5≤ C < 10 | 47 |
| 10≤ C< 15 | 85 |
| 15 ≤ C < 20 | 133 |
| 20 ≤C < 25 | 79 |
| 25 ≤C < 30 | 36 |
| 30 ≤C < 35 | 9 |

(a) Calculate the approximate mean and standard deviation of the cost
(b) Plot an Ogive for the above data. Use the Ogive to estimate the;
   (i)   median
   (ii)  range of the cost of the middle 60% of the books
   (iii) percentage of the text books which cost more than £ 16.

20. A particle of weight 30N rests on a rough plane inclined at an angle, θ to the horizontal where $n\theta = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle moves up the plane under the action of a force PN. Find the least value of P given that this force acts;
     (a)  Up the plane          (b)  horizontally

**END**