SENIOR SIX 2014 ONLINE SEMINAR (23RD APRIL – 15TH MAY 2014).

APPLIED MATHEMATICS P425/2

PLEASE ATTEMPT ANY QUESTION AND SEND IN YOUR SOLUTION TO ronaldddungu@yahoo.com

THEN WE SHALL COMPILE ALL THE SOLUTIONS AND RETURN TO YOU.

DESCRIPTIVE STATISTICS , SCATTER DIAGRAMS AND CORRELATION

1. A random sample of 97 people who own mobile phone was used to collect data on the amount of time they spent per day on their phones. The results are displayed in the table below.

Time spent per day, (t minutes)	$0 \leq t \langle 5$	$5 \leq t \langle 10$	$10 \leq t \langle 20$	$20 \leq t \langle 30$	$30 \leq t \langle 40$	$40 \leq t \langle 70$
Number of people	11	20	32	18	10	6

i) Calculate estimates of the mean and standard deviation of the time spent per day on these mobile phones.

ii) On graph paper, draw a fully labelled histogram to represent the data.

2. In an experiment to compare two methods of rearing real calves eight pairs of identical twins were used, one twin of each pair being allocated at random of each method of rearing. At the end of the experiments the calves were slaughtered and sample joints were cooked and scored for palatability with the following results:

Twin pair	…..1	2	3	4	5	6	7	8
Method A	27	37	31	38	29	35	41	37

Method B	23	28	30	32	27	29	36	31

(a) Calculate spearman's rank correlation coefficient

(b) Do the methods differ on the palatability score?

PROBABILITY AND PROBABILITY DISTRIBUTIONS

3. The people living in 3 houses are classified as children (*C*), parents (*P*) or grandparents (*G*). The numbers living in each house are shown in the table below:

House number 1	House number 2	House number 3
4C. 1P. 2G	*2C. 2P. 3G*	*1C. 1G*

i) All the people in all 3 houses meet for a party. One person at the party is chosen at random. Calculate the probability of choosing a grandparent.

ii) A house is chosen at random. Then a person in that house is chosen at random. Using a tree diagram, or otherwise, calculate the probability that the person chosen is a grandparent.

iii) Given that the person chosen by the method in part (ii) is a grandparent, calculate the probability that there is also a parent living in the house

4. A random variable X takes values in the interval $0 \leq x \leq 3$ and has probability density function

$$f(x), \text{ where } f(x) = \begin{cases} kx & 0 \leq x \langle 1 \\ ½k(3-x) & 1 \langle x \leq 3 \\ 0 & otherwise \end{cases}$$

(i) Find the value of k.

(ii) Find the expected value and the standard deviation of X.

(iii) Find the cumulative distribution function $F(x)$.

(iv) Find $P\left(x < 2 \middle/ x > 0.5 \right)$.

(v) Find $P\left(|x - 1| < \frac{1}{2} \right)$

5a) In an examination 30% of the candidates fail and 10% achieve distinction. Last year the pass mark (out of 200) was 84 and the minimum mark required for a distinction was 154. Assuming that the marks of the candidates were normally distributed, estimate the mean mark and the standard deviation.

b) A consumer group, interested in the mean fat content of a particular type of sausage, takes a random sample of 20 sausages and sends them away to be analysed. The percentage of fat in each sausage is as follows:

26 27 28 28 28 28 29 29 30 30 31 32 32 33 33 34 34 34 35 35

Assume that the percentage of fat is normally distributed with a mean μ, and that the standard deviation is known to be 3. Calculate a 98% confidence interval for the population mean percentage of fat.

6. Computer breakdowns occur randomly on average one in every 48 bought.

i) Calculate the probability that there will be fewer than 4 breakdowns in 60 computers bought.

ii) Find the probability that the number of breakdown in one year where 8760 computers are bought is more than 200.

iii) Independently of the computer breaking down, the computer sold is a factory damage on average two in every 24 computers sold. Find the probability that the total number of damaged computers and computer breakdowns in a 60 computer purchase by a certain school is exactly 30.

7. Machine *A* fills bags of fertilizer so that their weights follow a normal distribution with mean $20.05kg$ and standard deviation $0.15kg$. Machine *B* fills bags of fertilizer so that their weights follow a normal distribution with mean $20.05kg$ and standard deviation 0.27 kg.

i) Find the probability that the total weight of a random sample of 20bags filled by machine *A* is at least 2 kg more than the total weight of a random sample of 20 bags filled by machine *B*.

ii) A random sample of *n* bags filled by machine *A* is taken. The probability that the sample mean weight of the bags is greater than 20.07 kg is denoted by *p*. Find the value of *n*, given $p = 0.0250$ correct to 4 decimal places.

NUMERICAL METHODS

8a) Find the range within which the exact value of z lies, given that $Z = \frac{1}{x} + \frac{1}{y} + xy$, $x = 4.165 \pm 0.001,\ y = 6.72 \pm 0.01$, hence, calculate the percentage relative error in estimating Z.

b) Use the trapezium rule with 11 ordinates to find the approximate value of $\int_1^2 x\log_{10}x\,dx$ 4 d.p.

c) Find the exact value of $\int_1^2 x\log_{10}x\,dx$ .Hence calculate the error in (b) above. How can this error be reduced when using the trapezium rule.

9a) Show graphically that $\pi\sin x = x$ has three real roots.

b) Use linear interpolation to find the non zero roots to 1d.p

c) By constructing a table of values for $f(x) = 3xe^x - 1$ in the range $0.1 \le x \le 1.1$, obtain the root of $f(x)$ correct to 3d.p using the Newton-Raphson's formula.

10. Show that neither $X_{n+1} = \frac{2}{5}\left(4 - x_n^{\ 3}\right)$ nor $X_{n+1} = {(8-5x_n)}\big/{2x_n^{\ 2}}$ can be used to find the root of $2x^3 + 5x - 8 = 0$. Hence find the most suitable formula that can be used to find the root to 3decimal points.

MECHANICS

RELATIVE VELOCITY

11a) When a motorist is driving with velocity $6\mathbf{i} - 8\mathbf{j}$ the wind appears to come from the direction $\mathbf{i}$. When he doubles his velocity the wind appears to come from the direction $\mathbf{i} + \mathbf{j}$. Find the true velocity of the wind.

b) A ship A is travelling on a course of 060^o at a speed of $30\sqrt{3}km\,h^{-1}$ and a ship B is travelling on a course of 030^o at $20km\,h^{-1}$. At noon B is $260km$ due east of A. Find the time when A and B are closest together.

ACCELERATION

12a) A body initially at rest undergoes uniform acceleration. It covers a distance $2p$ in q seconds; and a distance $p-q$ in p seconds. Show that it covers a distance of $p+q$ in $\sqrt{(q^2-p^2)}$.

b) The force P acting along a rough inclined plane is just sufficient to maintain a body on a plane, the angle of friction λ being less than $\propto$, the angle of the plane. Find the least force, acting along the plane sufficient to drag the body up the plane.

c) A train takes a time T to perform a journey from rest to rest. It accelerates uniformly from rest for a time $\propto$T and retards uniformly to rest at the end of the journey for a time βT. During the intermediate time it travels with uniform speed V. Prove that the average speed for the journey is $\frac{1}{2}V(2-\propto-\beta)$.

13. A mass of $4kg$ rests on rough horizontal table (coefficient of friction $\frac{1}{2}$) and is connected by a light inextensible string, passing over a small smooth fixed pulley at the edge of the table to a smooth pulley of mass $\frac{1}{2}kg$ hanging freely. Over the hanging pulley passes a light inextensible string to the end of which are connected masses $2kg$ and $3kg$. If the system is released from rest find:

(i) the acceleration of the $3kg$ mass

(ii) the tension in the string passing over the hanging pulley

WORK, POWER, ENERGY AND MOMENTUM

14. A wooden block of mass $1.95kg$ initially at rest on a rough horizontal surface is hit by a bullet of mass $0.05kg$ travelling horizontally at $200m\,s^{-1}$. The bullet becomes embedded in the block. Find:

i) the initial common velocity of the bullet and block.

ii) the distance the block covers before it comes to rest given that the co-efficient of friction is 0.4.

iii) the velocity of the block-bullet system at the moment when it has covered a half of the distance in (ii) above.

15a) A light elastic string has natural length 2m and modulus of elasticity $200N$. The ends of the strings are attached to two fixed points P and Q which are on the same horizontal level $3m$ apart. An object is attached to the midpoint of the string and hangs in equilibrium at a point $0.5m$ below PQ. Calculate to 2 significant figures:

(i) The mass of the object.

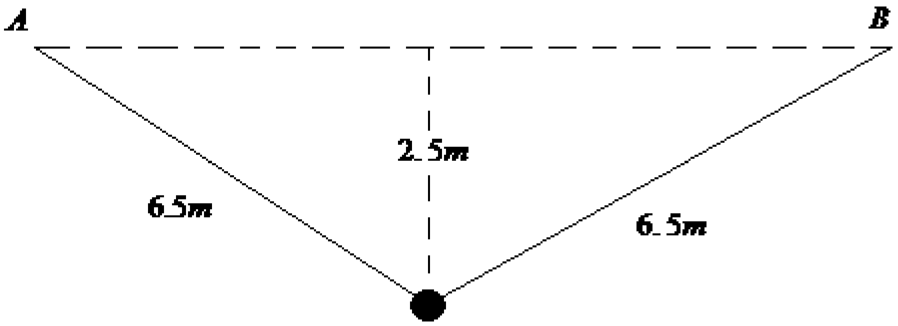
(ii) The elastic potential energy stored in the string in this position

16a) A force of magnitude $26N$ acting in the direction $3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$ causes a particle to undergo a displacement $6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. Find the work done by the force in moving the particle

b) A particle moves along a curve so that its position vector at time t is $\mathbf{r} = (3t - 2)\mathbf{i} + t^4\mathbf{j}$ metres. If one of the forces, acting on the particle is $\mathbf{F} = 5t^2\mathbf{i} - \mathbf{j}\ N$, find the power of F in terms of t. Hence find the work done by F in the interval $0 \leq t \leq 2$.

c) Forces $\mathbf{P} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{Q} = \mathbf{i} - 2\mathbf{j}$ act through points with position vectors 2**i**-**j** and 9**i**-2**j** respectively. Find the Cartesian equation of the line of action of the resultant of P and Q.

17.



A light elastic string has natural length $10m$ and modulus of elasticity $130N$. The end of the string are attached to fixed points A and B, which are at the same horizontal level. A small stone is attached to the mid - point of the string and hangs in equilibrium at a point 2.5 m below AB as shown in the diagram. With the stone in this position the length of the string is 13 m.

i) Find the tension in the string.

ii) Show that the mass of the stone is 3 kg.

The stone is now held at rest at a point 8 m vertically below the mid- point of AB.

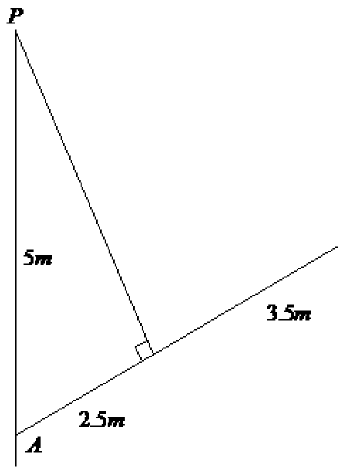
iii) Find the elastic potential energy of the string in this position.

iv) The stone is now released. Find the speed with which it passes through the mid-point of AB.

STATICS.

18. A uniform lamina of weight W is in the form of an isosceles triangle ABC with $AB = AC$. The midpoint of BC is E and D is the midpoint of AE. The triangle BDC is removed, if $BC = 2b$ and $AD = DE = d$, find the position of the centre of gravity of the remainder. A particle of weight W is attached at B and the resulting body suspended from A. show that AD is inclined to the vertical at an angle $\tan^{-1}\left(\frac{2b}{5d}\right)$.

19.



A uniform beam *AB* has length $6\ m$ and mass $45kg$. One end of a light inextensible rope is attached to the beam at the point 2.5 m from *A*. The other end of the rope is attached to a fixed point *P* on a vertical all. The beam is in equilibrium with A in contact with the wall at a point $5m$ below *P*. The rope is taut and at right angles to *AB* (see diagram). Find

i) the tension in the rope.

ii) the horizontal and vertical components of the force exerted by the wall on the beam at A.

PROJECTILES

20 a) A particle is projected vertically upwards with a velocity $u\ m\ s^{-1}$ after time, $t\ s$ another particle is projected vertically upwards from the same point and with the same initial velocity. Prove that they will collide at a point distance $\frac{4u^2 - g^2t^2}{8g}$ from the point of projection.

b) The heavy particle is projected from a point O at an angle of elevation α.Prove that the equation to trajectory is $y = x\left(1 - \frac{x}{r}\right)\tan\alpha$, where r is the horizontal range.

c) If the distance between two points on a path of the projectile which are at the same height, h above the horizontal is $2a$,show that $R(R - 4h\cot\alpha) = 4a^2$.

21. (a) A ball is projected so as to clear two walls, the first of height a, at a distance b from the point of projection and_the second of height b, at a distance a, from the point f projection. Show that the range on a horizontal plane is $\frac{a^2 + ab + b^2}{a + b}$

b) A ball is projected from a point on the ground distance a from the foot of a vertical wall of height b, the velocity of projection being V at an angle α to the horizontal. Find how high above the wall the ball passes it. If the ball just clears the wall prove that the greatest height reached is $\frac{1}{u}\left(\frac{a^2 \tan^2 \alpha}{a \tan \alpha - b}\right)$

c) Two particles are projected simultaneously with the same speed , v, in the same vertical plane prove that their velocities are parallel after a time $\frac{y}{g}\left(\frac{\cos \frac{\theta_1 - \theta_2}{2}}{\sin \frac{\theta_1 + \theta_2}{2}}\right)$