

P425/1

PURE MATHEMATICS

JULY 2015

Uganda Advanced Certificate of Education

INTERNAL MOCK EXAMINATIONS – 2015

PURE MATHEMATICS

P425/1

3 Hours

INSTRUCTIONS TO CANDIDATES

Answer **all** the eight questions in section **A** and any **five** from section **B**

Any additional question(s) answered will **not** be marked

All necessary working **must** be clearly shown

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks).

1. Given that $y = \frac{x+6}{\sqrt{(x+2)}}$ find the value of $\frac{dy}{dx}$ when $x=2$ (5 marks)

2. The population of Kampala increases at a rate of 5.6 % per annum. If the current population is 4 million. Find the population of Kampala

i. in 5 years time

ii. 10 years ago . (5 marks)

3. Expand $\frac{1-x}{(1+2x)^3}$ in ascending powers of x, up to and including the

term in x^4 and state the validity of x for which the expansion is valid.

(5marks)

4. The points A (1,2,3) , B (3,3,5) and C (3,0,5) form a triangle.

Calculate the area of this triangle. (5 marks)

5. Given that $y = \frac{\sin x}{x}$. Prove that $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + xy = 0$. (5 marks)

6. Evaluate $\int_{3m}^{4m} \frac{(x-m)}{(x^2-2mx)} dx$ (5 marks)

7. If $\sin A = \frac{15}{17}$ and angle A is acute, find the value of

$$\cos 2A + 3 \tan 2A \quad (5 \text{ marks})$$

8. Find the equation of a circle whose end diameter is the line joining the points $A(1, 3)$ and $B(-2, 5)$. (5 marks)

SECTION B (60 marks)

9. Given that $f(x) = \frac{x^3 + 2x^2 + 61}{(x+3)^2(4+x^2)}$. Express $f(x)$ into partial fractions. Hence find $\int f(x) dx$ and also find

$$\frac{d(f(x))}{dx} \quad (12 \text{ marks})$$

10. (a.) Given that $z = \frac{(1+2i)}{(3-i)^2}$ find the modulus and argument $\bar{z} + \frac{5}{z}$. (6 marks)

(b.) If $z = x + iy$, determine the Cartesian equation of the locus given by $\left| \frac{(z-1)}{(z+1-i)} \right| = \frac{2}{5}$. (6 marks)

11. (a.) Show that $\sin 4\theta = \frac{4t(t^2-1)}{t^4+2t^2+1}$, if $t = \cot \theta$. (5 marks)

(b.) Solve $10 \sin^2 x + 5 \sin 2x - \cos^2 x = 2$, for $0^\circ \leq \theta \leq 360^\circ$ (7 marks)

12. (a.) Differentiate e^{3x} from first principles. (4 marks)

(b.) Find $\int \frac{1}{(1-x)(1-x^2)^{\frac{1}{2}}} dx$ (4marks)

(c) Find $\int x^2 \cos 3x dx$ (4 marks)

13. P is the point $(ap^2, 2ap)$ on the parabola $y^2 = 4ax$ and OQ is the chord passing through the origin O and parallel to the tangent to the parabola at P. If the tangents to the parabola at P and Q meet at the point R.

(i.) Determine the coordinates of Q and R. Hence find the locus of R. (7 marks)

(ii.) Given that M is the midpoint of OQ, show that PMR is a right-angled triangle and find its area in terms of a and (5marks)

14. (a) Find the acute angle between the line $\frac{x-4}{2} = \frac{y+3}{-1} = \frac{1-z}{2}$ and the plane $6x + 2y - z = -4$ (6 marks)

(b) show that the lines $r = (-2i + 5j - 11k) + \alpha(3i + j + 3k)$ and $r = (8i + 9j) + t(4i + 2j + 5k)$ intersect hence find the position vector of their point of intersection. (6 marks)

15. (a.) Find n if ${}^n C_{14} = {}^n C_{16}$ (4marks)

(b.) Prove by induction that $5^n + 4n - 1$ is divisible by 8 for all positive integers. (4marks)

(c) What is smallest number of terms of a geometric progression (G.P) 5,10,20.... That can give a sum a greater than 800,000. (4marks)

16. (a.) Solve the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{x^2}$ (4 marks)

(b) The temperature of a cooling body is known to decrease at a rate proportional to the temperature difference of the body and the surrounding. A police man found a normal body of a man lying on the road side at 6:00 am. The temperature of the body and the surrounding were 33°c and 18°c respectively. At 7:00am, the temperature of the body and surrounding were found to be 28°c and 18°c respectively. If the normal temperature of the human being is known to be 37°c .

(i.) Write down a differential equation to represent the above information and solve it.

(ii.) Estimate the time when the man was killed. (8 marks)

THE END.