UNDERSTANDING CIRCLE THEOREMS-PART ONE.

Common terms:

(a) **ARC**- Any portion of a circumference of a circle.

(b) **CHORD**- A line that crosses a circle from one point to another. If this chord passes through the centre then it is referred to as a diameter

(c) **A TANGENT**- A line that touches a circle at only one point.

Theorem 1.

The angle subtended at the centre of a circle is twice the angle subtended at the circumference by the same arc.

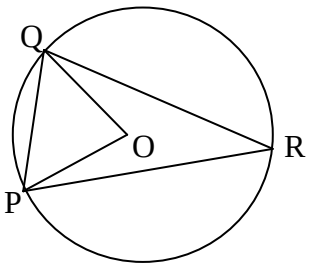
Theorem 2.

Angles subtended by an arc in the same segment of a circle are equal.

Example 1.

Given PQO = 65^0

Find QRP



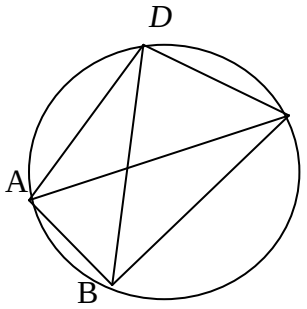
Triangle OQP is isosceles (OP = OQ, the radii)

:. OPQ = 65^0 :. QOP = $180^0 - (65^0 + 65^0)$ (angle sum of a triangle)

= 50^0

:. QRP = 25^0 (half of angle at the centre).

Example2.



Given that angle BDC = 78^0 and DCA = 56^0.

Find angles BAC and DBA.

Solution: BAC = BDC = 78^0. (both subtended by arc BC)

DBA = DCA = 56^0. (both subtended by arc AD)

Theorem3.

The opposite angles in a cyclic quadrilateral add up to 180^0 (the angles are supplementary).

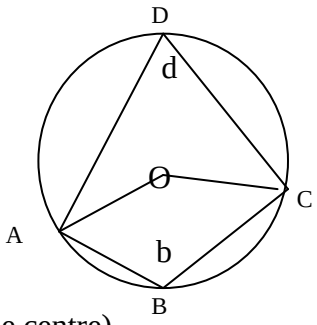
ABCD is a **cyclic quadrilateral** because all its vertices touch the circumference of the circle.(ABCO is not cyclic because O is not at the circumference).

Proof:

OA and OC are radii.

Let angle ADC = d

and angle ABC = b



AOC obtuse = 2d (angles at the centre)

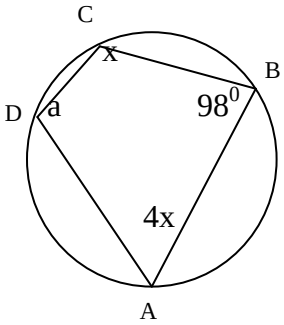
AOC reflex = 2b (angles at the centre)

:. $2d + 2b = 360^0$ (angles at a point)

:. $d + b = 180^0$ as required

Example3.

Find *a* and *x*



CDA = a, ABC = 98^0 , DCB = x^0 ,DAB = $4x^0$

a = $180^0 - 98^0$ (opposite angles of a cyclic quadrilateral)

:. $a = 82^0$

$x + 4x = 180^0$ (opposite angles of a cyclic quadrilateral

$5x = 180^0$

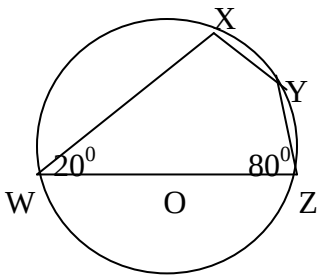
$x = 72^0$

Exercise. 1.ABCD is a quadrilateral inscribed in circle, centre O, and AD is a diameter of the circle. If angle CDB = 46^0 and ADB = 31^0. Calculate

(a) the angle ABC (b) the angle BCD (c) the angle BAD.

2. A circle has a radius of 155mm .AB is a chord of this circle which is 275mm long. What angle does AB subtend at the circumference of the circle.

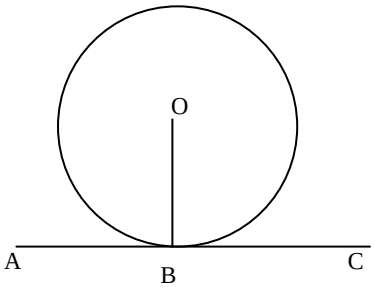
3. Given angle XWZ = 20^0 , angle WZY = 80^0 and O is the centre of the circle
   (a) Find angle WXY
   (b) Show that WY bisects XWZ



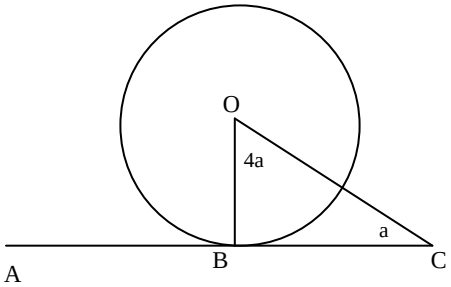
Theorem 4.

The angle between a tangent and the radius drawn to the point of contact is 90^0

Line ABC is a tangent and angle ABO = 90^0



Example 4. Find the angle BCO and angle BOC.



$4a + a + 90^0 = 180^0$

$5a = 90^0$

$a = 18^0$.

Angle BCO = 18^0 and BOC = 4 x 18^0 = 72^0.