PROBABILITY

1.0 <u>Chance events</u>:

In this chapter, we shall be concerned with chance events. For example, a weatherman makes a forecast of the future weather. His forecast, "Rain" is more accurately a probability statement.

"The Express Football team will win the Super division league". But what you mean to say is, "It is likely that express will win the league."

Probability has many practical uses. For example, the school uses probability in setting up budget requirements; Politicians use probability to work out their strategies in order to win elections.

In this section, we study some ideas about statements involving chance events, like "if I toss a coin and allow it to fall freely, it will show heads" or "Gayaza will win the Schools Lawn tennis Championships".
We will concern ourselves with a measure of chance that an event will happen. This measure of chance is also called the <u>probability</u> that the event will occur.

1.1 <u>Definition:</u>

Probability is a measure of the likelihood of a required outcome happening. It is usually given as a fraction.

$$\text{Probability} = \frac{\text{number of required outcomes}}{\text{Number of possible outcomes.}}$$

Example 1

If a coin is tossed once, find the probability that a head appears.

Possible outcomes = { Head, tail }

Required outcomes = { Head }

Therefore probability that head appears, denoted as P(Head) = $^1/_2$

Example 2: A die is rolled once (i) Find the probability that an even number appears.

Possible outcomes = {1, 2, 3, 4, 5, 6}

Required outcomes, "set of even numbers"

= { 2, 4, 6}

P(even numbers) = $\frac{3}{6} = \frac{1}{2}$

(ii) Find the probability that a prime number appears

required outcome, "set of prime numbers"

= { 2, 3, 5 }

P(Prime) = $\frac{3}{6} = \frac{1}{2}$

(iii) Find the probability that an even prime number appears.

Set of even prime numbers = { 2 }

P(even prime number) = $\frac{1}{6}$.

Example 3:

If there are n equally likely ways of doing a certain action and x of them produce a certain event A, then the theoretical probability of A happening is given by

$$P(A) = \frac{\text{number of ways in which A happens}}{\text{Number of ways in which the action is done.}}$$

$$= \frac{x}{n}.$$

The probability that event. A will not happen, denoted

$$P(A') = \frac{n - x}{n}$$

Since there are n – x ways in which A does not happen,

Note: A' means A does not happen and is read as A compliment.

$$\therefore P(A') = \frac{n - x}{n}$$

$$= 1 - \frac{x}{n}$$

but $\frac{x}{n} = P(A)$

$$\therefore P(A') = 1 - P(A)$$

□ $$P(A') + P(A) = 1$$

Which is a certainty since A must either happen or not happen.

If an event cannot possibly happen, then its probability = 0 (impossibility). For the probability of picking a white ball from a bag containing black balls only is 0. But the probability of picking a black ball is 1.

Therefore if P is the probability of an event happening, then P lies in the range $0 \leq P \leq 1$.

Example 4:

Sarah and Edith have played each other at tennis 15 times this season. Sarah has won 12 of the matches. They play each other in a championship. What is the probability that

(a) the match is drawn?

(b) Sarah wins?

(c) Either Sarah or Edith wins?

Solution:

Tennis matches are either won or lost. They are never drawn.

(a) P(draw) = 0

(b) Sarah has won 12 of the last 15 matches

$$P(\text{Sarah winning}) = \frac{12}{15}$$

$$= 0.8$$

This is an example of experimental probability. It is based on the number of matches won out of the total played.

(c) Since one of either Sarah or Edith must win, the probability of either person winning = 1

Exercise:

1. A letter is chosen at random from the alphabet. Find the probability that it is:

a) B

b) L or T

c) One of the letters of the word BETHEL

d) Not one of the letters of the word DIRECT

Note: "at random" means "in a free and irregular way"

1.2 . MUTUALLY EXCLUSIVE EVENTS.

If the events A and B cannot happen at the same time, then they are said to be mutually exclusive events. That is to say an event (A) excludes the possibility of the other event (B) or the other way round.

Addition law.

If the events A, B, C, ……, are mutually exclusive, the probability of A or B or C or …. happening is the sum of their individual probabilities.

$P(A \text{ or } B \text{ or } C \text{ or } \ldots) = P(A) + P(B) + P(C) + \ldots..$

$= P(A \cup B \cup C \cup \ldots)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) - P(A \cap B \cap C)$

Example 1:

Find the probability that a letter chosen at random from the alphabet is either a vowel or one of the letters P, Q, R, S. Consider the 2 sets;

Vowels, V = { A, E, I, O, U}   B = { P, Q, R, S}

$\therefore P(V \text{ or } B) = P(V) + P(B) = \frac{5}{26} + \frac{4}{26} = \frac{9}{26}.$

$= P(V \cup B) = P(V) + P(B) - P(V \cap B)$, but $P(V \cap B) = 0$

Note: The set of possible outcomes is also called the **sample space**.

Example:

What is the sample space of an expectant mother?

Sample space = {baby boy, baby girl }

1.3 Independent events.

If two events A and B can happen at the same time then they are said to be independent events. In such cases, the separate probabilities are multiplied to give a combined probability.

Product law.

If events A, B, C are independent, then the probability of A and B and C happening is the product of their individual probabilities.

$P(A \text{ and } B \text{ and } C \ldots) = P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

Example:

A bag contains 7 black balls and 5 white balls. A ball is drawn from the bag and replaced, and then a second one is drawn. What is the probability that: a) one is black and one is white.

b) at least one is black?

Solution. There are 12 balls of which 7 are black.

$$P(\text{drawing a black}) = \frac{7}{12}.$$

Since the first ball is replaced, then at the second choice, there are 12 balls of which 5 are white.

$$P(\text{drawing a white}) = \frac{5}{12}.$$

The second colour drawn is independent of the first colour drawn.

∴Probability of drawing first a black ball and then a white ball

$$= \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$$

But these two cases are mutually exclusive events;

∴ Probability of drawing a black ball and a white ball when the order does not matter yields P(BW or WB)

$$= \frac{35}{144} + \frac{35}{144} = \frac{70}{144} = \frac{35}{72}$$

b) Probability of at least one black ball

= 1 – probability of no black ball.

$$= 1 - \frac{5}{12} \times \frac{5}{12}$$

$$= 1 - \frac{25}{144.} = \frac{119}{144}$$

Notice that the product law is used to solve problems, which contain the word and .

1.4 Outcome table, tree diagrams(sample space).

Example: Two dice are thrown at the same time. Find the probability of getting:

a) at least one 4

b) a total score which is prime.

c) Probability of getting a score of (i) 3 (ii) 5 (iii) 7 (iv) 11

(A score refers to the sum of the numbers showing on the two dice)

The table below shows all the possible outcomes when two dice are thrown.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	**5**	6	7
2	3	4	5	**6**	7	8
3	4	5	6	**7**	8	9
4	**5**	**6**	**7**	**8**	**9**	**10**
5	6	7	8	**9**	10	11
6	7	8	9	**10**	11	12

Solution:

(a) P(at least one 4) = P(one or two 4's) = 11/36.

(b) P(prime score) = P(2,3,5,7,11) = 15/36

= 5/12.

(c) P(score of 3) = 2/36 =1/18.

(d)P(score of 5) = 4/36 = 1/9.

(e) P(score of 7) = 6/36 = 1/6.

(f) P(score of 11) = 2/36 = 1/18.

Note:

For one die, there are 6 outcomes.

For the two dice, there are 6^2 = 36 outcomes.

For three dice, there are 6^3 outcomes

Also for a coin, there are 2 outcomes

For two coins, 2^2 = 4 outcomes

	H	**T**
H	H,H	H,T
T	T,H	T,T

And for 3 coins, 2^3 = 8 outcomes.

In the experiments of tossing a coin and throwing a die, the symmetry of a coin tells us that "heads" and "tails" should come down about equal number of times.

P(head) = P(tail) = 1/2

Also for a fair die, all the numbers should show up about equal number of times.

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.

We say that these events are equally likely. Probability found by such an argument is called **theoretical probability.**

TREE DIAGRAMS.

Example A bag contains 3 black balls (B) and 2 white balls (W).

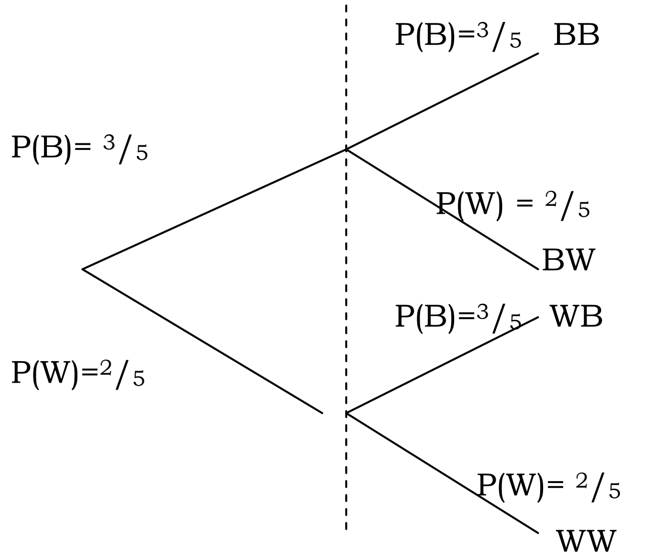
a) A ball is taken from the bag and then replaced. A second ball is chosen. What is the probability that

(i) they are both black?

(ii) One is black and one is white?

b) Find out how those probabilities are affected if two balls are chosen without any replacement.

a) (i) With replacement.



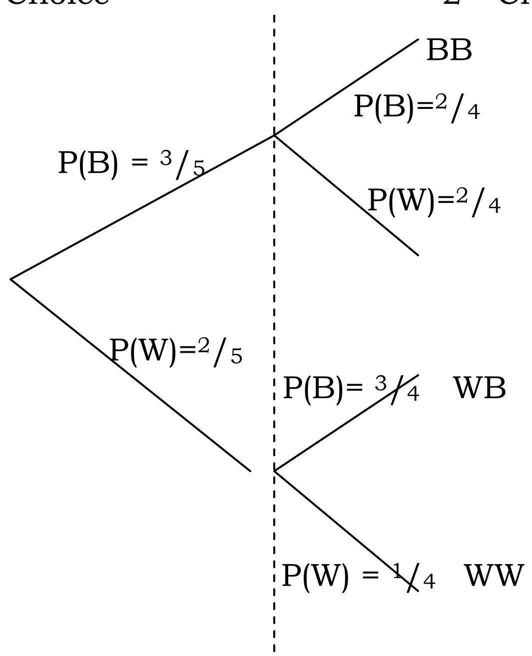
1st choice 2nd choice.

Solution: (i)P(BB) = $^3/_5 \times {}^3/_5 = 9/25$.

(ii) P (BW or WB) = $^3/_5 \times {}^2/_5 + {}^2/_5 \times {}^3/_5 = 12/25$.

b) (ii) Without replacement.

1st Choice 2nd Choice



Solution: (i) P(BB) = 3/5 X 2/4 = 6/20 = 3/10.

(ii) P (BW or WB) = 3/5 X 2/4 + 2/5 X 3/4 = 12/20 =3/5.

Further examples:

(1) If the probability of student A solving a problem is 6/7, of student B solving it is 7/8 and of student C solving is 8/9. Find the probability that:

a) they will not solve it.

P(A fails and B fails and C fails) = 1/7 x 1/8 x 1/9 = 1/504.

b) at least one will solve it.

P(one or two or three solve it) = 1 – P(none solves it).

= 1 – 1/504

= 503/504.

Exercise.

1. If two dice are thrown, find the probability of getting

a) an odd score b) a score more than 7

2. A bag contains 4 red marbles and 5 blue marbles. If one marble is picked, the colour noted and replaced and then another marble is picked. What is the probability of (a) both being red (b) one of each colour.

3.When 4 coins are tossed, what is the probability of

a) 4 heads

b) 2 heads and 2 tails

c) at least one head.

4. The ratio of red to yellow to green balls in a large bag is 5:4:6. Two balls are picked at random. Find the probability that:

(a) they are both green (b) they are of different colours

5. A box of eggs contains 9 good ones and 3 cracked ones. Find the probability that out of 3 eggs from the box,

a) they are all good b) one is good and 2 are cracked.

6. In a class of 100 students, 41 take maths, 29 take geography, 28 take economics, 15 take maths and geography, 8 take geography and economics, 19 take maths and economics and 5 take all the three subjects. Using a Venn diagram, find the probability that a student selected at random:

(a) takes none of the three subjects.

(b) Find the probability that a student selected at random takes at least 2 subjects.

7. The frequency distribution below shows the marks obtained in mathematics examination by 2000 candidates.

Mark	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	30	60	220	540	490	310	180	110	60

Find the probability that a student selected at random scored:

(a) above 60%

(b) between 30% and 81%

(c) 75% and above.