Name: ……………………………………………………………………. Date ………………………………………..

GAYAZA HIGH SCHOOL

SOLUTIONS FOR S.2 MATH WORKSHEET FIVE ON SETS

PREREQUISITE KNOWLEDGE:

- NUMBERS
- INTERGERS

SETS PART II

RECALL:

OPERATIONS ON SETS

Given sets A and B, we can define the following operations:

Operation	Notation	Meaning
Intersection	$A \cap B$	all elements which are in both A and B
Union	$A \cup B$	all elements which are in either A or B (or both)
Difference	$A - B$	all elements which are in A but not in B
Complement	$\overline{A}\ (or\ A^c\ )$	all elements which are not in A

Example 1:	Example 2:
Let $A = \{1,2,3,4\}$ and let $B = \{3,4,5,6\}$. Then: $A \cap B = \{3,4\}$ $A \cup B = \{1,2,3,4,5,6\}$ $A - B = \{1,2\}$ $A^c = \{5, 6\}$	Let $A = \{y, z\}$ and let $B = \{x, y, z\}$. Then: $A \cap B = \{y, z\}$ $A \cup B = \{x, y, z\}$ $A - B = \emptyset$ $A^c = \{x\}$

1. Find the union of each of the following pairs of sets.

(a) **A** = {2, 4, 6}, **B** = {1, 2, 3} $\boldsymbol{A \cup B} = \{\mathbf{1, 2, 3, 4, 6}\}$	(b) **P** = {a, e, i, o, u}, **Q** = {a, b, c, d} $\boldsymbol{P \cup Q} = \{\mathbf{a, b, c, d, e, i, o, u}\}$
(c) **X** = {x: n ∈ N, x = 2n, n < 4} **Y** = {x: x is an even number less than 10} **X** = {2, 4, 6} and **Y** = {2, 4, 6, 8} $\boldsymbol{X \cup Y} = \{\mathbf{2, 4, 6, 8}\}$	(d) $\boldsymbol{M} = \{x: x$ is natural number and multiple of 3} $\boldsymbol{N} = \{x: x$ is a prime number less than 19} **M** = {3, 6, 9, 12, 15, 18…} and **N** = {2, 3, 5, 7, 11, 13, 17} **M ∪ N = {2, 3, 5, 7, 9, 11, 13, 17, 6, 9, 12, 15, …}**

2. Find the intersection of each of the following pairs of sets.

(a) **A** = {1, 4, 9, 16} and **B** = {3, 6, 9, 12} $\boldsymbol{A \cap B} = \{\mathbf{9}\}$	(b) **C** = {p, q, r, s} and **D** = {a, b} **C ∩ D** = {}
(c) **P** = {x : n ∈ N, x = 3n n< 3} **Q** = {x : x ∈ N x < 7} **P** = {3, 6} and **Q** = {1, 2, 3, 4, 5, 6} $\boldsymbol{P \cap Q} = \{\mathbf{3, 6}\}$	(d) **X** = {x : x is a letter of the word '**LOYAL**'} **Y** = {x : x is a letter in the word '**FLOW**'} $\boldsymbol{X \cap Y} = \{\mathbf{L, O}\}$

3. If A = {a, b, c, d}, B = {b, c, d, e}, C = {c, d, e, f}, D = {d, e, f, g}, find

(a) A - B = **{a}**	(b) B – C = **{b}**
(c) C - D = **{c}**	(d) D - A = **{e, f, g}**

4. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 4, 6, 8, 10}, B = {1, 3, 5, 7, 8, 9}
Find:

(a) A' = **{3, 5, 7, 9}**	(b) B' = **{2, 4, 6, 10}**
(c) B' = **{2, 3, 4, 5, 6, 7, 9, 10}**	(d) A' ∪ B'= **{2, 3, 4, 5, 6}**
(e) A' ∩ B'= **{1, 2, 3, 4, 5}**	(f) (A ∪ B)'= **{∅}**

5. Find the complement of the following sets if universal set is the set of natural numbers.

(a) { : x is a prime number} **{x: x is composite number and 1}**	(b) {x: x is a multiple of 2} **{x: x is odd}**
(c) {x: x is a perfect cube} **{x: x is not a perfect cube}**	(d) {x: $x \geq 10$} **{x: $x < 10, x \in N$}**
(e) {x: $x \in N, 5x + 1 > 20$} **{x : $x \in N$ and $x < 4$}**	(f) {x: x is an odd natural number} **{x: x is even}**

6. If U = {a, b, c, d, e, f} find the complement of the following.

(a) **A** = { } **A' = U**	(b) **B** = {c, d, f} **B' = {a, b, e}**
(c) **D** = {a, b, c, d, e, f} **D' = ∅**	(d) **C** = {a, b, d} **C' = {c, e, f}**
(e) **E** = {b, c} **C' = {a, d, e, f}**	(f) **F** = {a, c, f} **F' = {b, d, e}**

7. If U = {1, 2, 3, 4, 5, 6} and A = {2, 3, 6}, find

(a) $A \cup A' =$ **U**	(b) $\emptyset \cap A =$ **A**
(c) $A \cap A' = \emptyset$	(d) $U' \cap A = \emptyset$

RECALL
SUBSETS
A set **A** is a subset of a set **B** if every element in **A** is also in **B.**
For example, if $\boldsymbol{A} = \{1,3,5\}$ and $\boldsymbol{B} = \{1,2,3,4,5\}$, then **A** is a subset of **B** , and we write
$\boldsymbol{A} \subseteq \boldsymbol{B}$
The line under the sideways ∪ means that **A** may also be equal to **B** (that is, they may be identical sets).
If we want to say that **A** is a proper subset of **B** (that means: it's a subset, but there is at least one element in **B** that is not in **A**) then we can remove the line: **A⊂B**

To write that a set is not a subset of another set, just put a slash through the sideways ∪:
B⊄A

NOTE:
(a) **Number of Subsets of a given Set:**
If a set contains ‘n’ elements, then the number of subsets of the set is $\boldsymbol{2^n}$.

(b) **Number of Proper Subsets of the Set:**
If a set contains ‘n’ elements, then the number of **proper subsets** of the set is $\boldsymbol{2^n - 1}$.
If $A = \{p, q\}$ the proper subsets of A are [{ }, $\{p\}, \{q\}$]
⇒ Number of proper subsets of A are $3 = 2^2 - 1 = 4 - 1 = 3$.
In general, number of proper subsets of a given set = $2^n - 1$, where n is the number of elements.

NB: When we work with sets, we often have to define something called the **universal set** - this is the set of all things we are interested in. All of the sets that we work with must have elements in this universe - therefore, **each set we work with is a subset of the universal set.**

8. If A {1, 3, 5}, then write all the possible subsets of A. Find their numbers.

The subset of A containing no elements - { }
The subset of A containing one element each - {1} {3} {5}
The subset of A containing two elements each - {1, 3} {1, 5} {3, 5}
The subset of A containing three elements - {1, 3, 5)

Therefore, all possible subsets of A are { }, {1}, {3}, {5}, {1, 3}, {3, 5}, {1, 3, 5}
Therefore, number of all possible subsets of A is 8 which is equal 2^3.
Proper subsets are = { }, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}
Number of proper subsets are 7 = 8 - 1 = $2^3 - 1$

9. If the number of elements in a set is 2, find the number of subsets and proper subsets.

Number of elements in a set = 2

Then, number of subsets = $2^2 = 4$

The number of proper subsets = 2^3 - 1

= 4 – 1 = 3

10. If A = {1, 2, 3, 4, 5} then the number of subsets and proper subsets is?

Number of subsets $= 2^5 = 32$

The number of proper subsets $= 2^n - 1$

$= 2^5 - 1$

$= 32 - 1$

$= 31$

11. 40 students were surveyed about their favorite gospel singer.

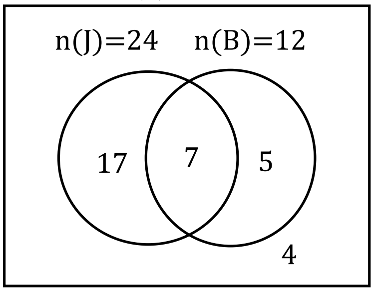
- 24 liked Judith.
- 12 liked Bugembe.
- 7 liked both. How many students like one or the other?

Solution

Let J denote Judith

Let B represent Bugembe

n(ξ)=40



$n(J \cup B) = 17 + 7 + 12 = 36$

12. Odel has just called you in to solve a difficult mystery. A group of thieves are getting ready to rob the bank, but he does not know exactly how many will do the job.

- 17 of them are known safecrackers.
- 28 of them are known getaway drivers.
- There are 40 thieves who are safecrackers and/or drivers.

Odel knows that the group that will rob the bank will be those thieves who are both safecrackers and getaway drivers, but he didn't pay attention in math class and can't figure it out. How many people will be in this group?

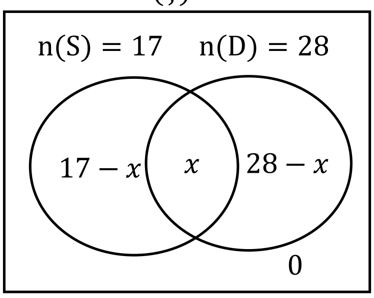
Solution:

Let S represent safecrackers

D represent gateway drivers

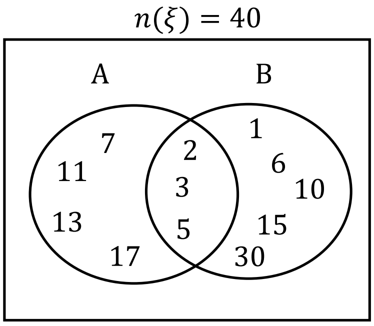
Let x be the number of thieves who are both safecrackers and getaway drivers

$n(\xi) = 40$



$$17 - x + x + 28 - x = 40$$
$$17 + 28 - x = 40$$
$$45 - x = 40$$
$$x = 45 - 40$$
$$x = 5$$

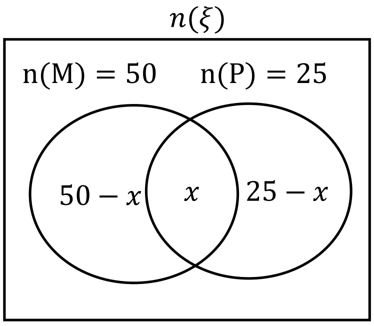
13. Given that $A = \{2, 3, 5, 7, 11, 13, 17\}$ and $8 = \{1, 2, 3, 5, 6, 10, 15, 30\}$. Find n(A ∩B)'



n(A ∩B)'= {1, 6, 7, 10, 11, 13, 15, 17, 30}

14. The sets M and P are such that $n(M) = 50, n(P) = 25$ and $n(M \cup P) = 60$. Calculate $n(P \cap M)$.

Let $n(P \cap M) = x$



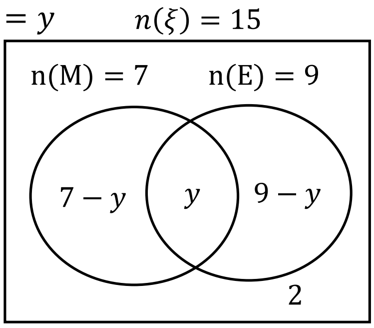
$$50 - x + x + 25 - x = 60$$
$$50 + 25 - x = 60$$
$$75 - x = 60$$
$$x = 75 - 60$$
$$x = 15$$

15. In a class of 15 students, 7 like Mathematics, 9 like English and 2 like neither Mathematics nor English. Find the number of students who like both Mathematics and English

Let M represent Mathematics

E represent English

$n(M \cap E) = y$



$$7 - y + y + 9 - y + 2 = 15$$
$$7 + 9 + 2 - y = 15$$
$$18 - y = 15$$
$$y = 18 - 15$$
$$y = 3$$

16. Given that $M = \{the\ first\ five\ multiples\ of\ 3\}$ and $S = \{the\ first\ five\ square\ numbers\}$, find;

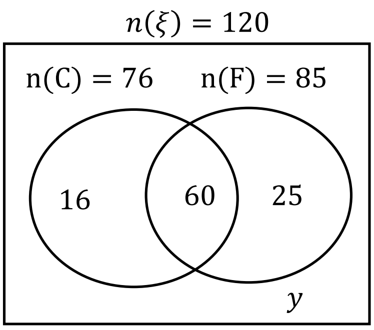
(a) $M \cap S$

$$M = \{3, 6, 9, 12, 15\}$$
$$S = \{1, 4, 9, 16, 25\}$$

(a) $M \cap S = \{9\}$

(b) $n(M \cap S) = 1$

17. A senior three class had 120 girls. 76 opted to take Commerce (C). 25 took French (F) only. 60 girls took both F and C. How many girls took neither C nor F.



Let y the number of girls who took neither C nor F

$$16 + 60 + 25 + y = 120$$
$$101 + y = 120$$
$$y = 120 - 101$$
$$y = 19$$

END.