

PROBABILITY REVISION QUESTIONS

1. A discrete random variable Y has the probability distribution function given by;

| | | | | | |
|----------|---|-----|---|-----|-----|
| y | 5 | 8 | 9 | 11 | 12 |
| $P(Y=y)$ | a | 0.1 | a | 0.4 | 0.1 |

Find the; (i) value of a (ii) $E(5Y-7)$ (iii) $\text{Var}(3Y)$

2. A discrete random variable X has a pdf, $f(x)$ given by; $f(x) = \begin{cases} k(x+1), & x = 1,2,3,4 \\ kx, & x = 5,6,7 \\ 0, & \text{elsewhere} \end{cases}$

Find the;

- a) value of a constant, k
 - b) $P(2 \leq x \leq 7)$
 - c) mean and mode of X
 - d) standard deviation of X
 - e) semi inter-quartile range
3. Awilo played 15 chess games. The probability that she wins a game is 0.6.
- (i) Find the probability that she won exactly 6 games
 - (ii) Calculate the most likely number of games she won.
4. A random variable X has the following probability distribution

$$P(X = -2) = P(X = -1) = 2P(X = 0), \quad P(X = 1) = 0.2, \quad 2P(X = 2) = P(X = 3) = 0.3$$

Find the;

- a. $P(X = 2)$ and $P(X = 0)$
 - b. $P(x \leq 2/x \geq 0)$
 - c. standard deviation of X
 - d. upper quartile
5. A student answers 3 questions. The chance of getting each question correct is $\frac{2}{3}$. Find the:
- (i) probability distribution for the number of correct answers
 - (ii) probability of obtaining at most 2 correct answers.
6. A biased coin is such that a head is three times as likely to occur as a tail, if its tossed five times. Find the probability that
- (i) at least two heads occur
 - (ii) exactly three tails occur.
7. A and B are two independent events with A thrice as likely to occur as B. Find
- (i) $P(A \cup B)$ (ii) $P(A/\bar{B})$

8. Two events A and B are independent such that $P(A \cup B) = 5P(B) = 4P(A)$. Determine $P(A)$ and $P(A \cap B)$
9. Events A and B are such that $P(A) = \frac{4}{7}$, $P(A \cap \bar{B}) = \frac{1}{3}$ and $P(A/B) = \frac{5}{14}$ Find
 (i) $P(A \cap B)$ (ii) $P(B)$
10. A and B are two independent events with A twice as likely to occur as B. If $P(A) = \frac{1}{2}$, find
 (i) $P(A \text{ or } B \text{ but not both})$ (ii) $P(A/B')$
11. In a certain inter university tournament; **35%** watched football but not cricket, **10%** watched cricket but not football and **40%** did not watch either game. Find the probability that they watched football, given that they did not watch cricket.
12. The probability that it will be sunny tomorrow is 0.25. The probability that Mrs Black will go shopping tomorrow is 0.6. The probability that both these events occur is 0.15. Find the probability that;
 (i) Mrs Black goes shopping given that it is not sunny.
 (ii) it is sunny given that Mrs Black goes shopping.
13. A certain mechanism consists of the three components and will operate properly only if all three components are functioning. Assuming that the three components function independently of one another and have probabilities 0.02, 0.05 and 0.10 respectively of developing a fault.
 (a) Find the probability that the mechanism will not operate properly
 (b) Given that the mechanism is not operating, find the probability that it is because exactly one of the components have developed a fault.
14. A and B are mutually exclusive events such that $P(A) = 0.5$ and $P(B) = 0.4$. Find
 (i) $P(A' \cap B')$ (ii) $P(A' \cup B)$
15. Mark, Mary and Mable applied for a job in a company. Their respective probabilities of getting a job were 0.8, 0.7 and 0.6. find the probability that
 (a) none of them got a job
 (b) at least one of them got a job.

16. (a) Given that A and B are two events such that $P(A')=0.3$, $P(B) = 0.1$ and $P(A/B')= 0.2$.
Find (i) $P(A \cup B)$ (ii) $P((A/B'))$

(b) A box contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

17. Three machines A, B and C produce solar bulbs in the ratio 30%, 60% and 10%. Of those produced by machine A, 25% are coloured, that of B is 30% and that of C is 70%. Find the probability that a bulb selected at random is;
- (i) coloured
(ii) produced by C given that is not coloured.
18. A random variable t takes values in the interval $0 < t < 3$ and has probability density function $f(t)$ given by

$$f(t) = \begin{cases} at; & 0 \leq t \leq 1 \\ \frac{a}{2}(3-t); & 1 \leq t \leq 3 \\ 0; & elsewhere \end{cases}$$

Where a is a constant, find the

- (i) value of a
(ii) expected value and variance of T
(iii) determine the distribution function of T , hence find $P(|T - 1| < \frac{1}{2})$
19. The probability density function of a random variable X is given by;

$$f(x) = \begin{cases} \frac{4}{5}x & ; 0 < x < 1 \\ \frac{2}{5}(3 - x) & ; 1 < x < 2 \\ 0 & ; otherwise \end{cases}$$

- (a) Sketch the function $f(x)$ and show that the area = 1
(b) Find the mean of x
(c) Determine the cumulative distribution function $F(x)$
20. A probability density function is given as;

$$f(x) = \begin{cases} kx(4 - x^2); & 0 \leq x \leq 2 \\ 0 & elsewhere \end{cases}$$

- Find (i) the value of k
(ii) mode
(iii) mean
(iv) standard deviation

END