P425/2

S6 NUMERICAL METHODS

Time : 1 hour

1 Use trapezium rule with 5 strips to evaluate $\int_0^2 x^2 e^{-x} dx$, correct to three decimal places.

05marks

2 (a) Use the trapezium rule with 7 ordinates to estimate the value of $\int_{0.5}^{1} \frac{x^2}{1+x^2}\, dx$, correct to 4 decimal places. *06marks*

(b) Calculate the percentage error in using the trapezium rule to estimate the integral in (a) above correct to 2 significant figures. *06marks*

3 The table below is an extract from the tables of $cosecx^{\circ}$

x	$12'$	$18'$	$24'$	$30'$	
$cosec60°$	1.1524	1.1512	1.1501	1.1490	

Use linear interpolation or extrapolation to estimate

(a) $cosec\ 60°\ 10'$ (b) $cosec^{-1}1.1497$ *05marks*

4 The height of a seedling of a given species is found to vary constantly with time. At the end of the first, second, third and fourth weeks, the heights are approximately 4.51cm, 4.92cm, 5.10cm and 5.36cm respectively.

(i) find the height of the seedling at the end of the fifth week

(ii) after how many hours will the seedling be 4.86cm? *05marks*

5 The radius and height of a cylinder was measured and found to be 5cm and 10cm with errors $\pm 0.2cm$ and $\pm 0.5cm$ respectively. Find the percentage error made in the calculation of the volume of the cylinder. *05marks*

6 (a) The numbers x, y and z were estimated with errors $\Delta x, \Delta y$ and Δz respectively. Show that the maximum relative error in $\frac{xy^2}{z}$ is $\left|\frac{\Delta x}{x}\right| + 2\left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right|$. State the assumptions made.

(b) If the maximum possible percentage errors in x, y and z are 2,3 and 4 respectively, calculate the maximum possible percentage error in $\frac{xy^2}{z}$. *12marks*

Qn		marks

1.

$$h = \frac{2-0}{5} = \frac{2}{5} = 0.4$$

B1 for h

x	Extreme values	Middle values
0	0	
0.4		0.1073
0.8		0.2876
1.2		0.4337
1.6		0.5169
2	0.5413	
totals	0.5413	1.3455

B1 *for all values of x*
B1 *for all values of y*

$$\therefore \int_0^2 x^2 e^{-x} dx \approx \frac{1}{2} \times \frac{2}{5} [0.5413 + 2 \times 1.3455]$$

$$\approx 0.646$$

M1 *iff ≈ seen*

A1 *cao 3dps*

05

2.

$$h = \frac{1-0.5}{7-1} = \frac{1}{12}$$

B1 for h

x	Extreme values	Middle values
$\frac{1}{2}$	0.20000	
$\frac{7}{12}$		0.25389
$\frac{2}{3}$		0.30769
$\frac{3}{4}$		0.36000
$\frac{5}{6}$		0.40984
$\frac{11}{12}$		0.45660
1	0.50000	
totals	0.70000	1.7880

B1 *for all values of x*
B1 *for all values of y*

$$\therefore \int_{0.5}^1 \frac{x^2}{1+x^2} dx \approx \frac{1}{24}[0.70000 + 2 \times 1.7880]$$

$$\approx 0.17817 \approx 0.1782 \text{ (4dp)}$$

M1 *iff≈ seen*

A1 *cao 4dps*

Exact value = $\int_{0.5}^{1} \frac{x^2}{1+x^2}\ dx$

$(x^2+1)\sqrt{x^2}$

$-\quad (x^2+1)$

-1

Exact value = $\int_{0.5}^{1} \frac{x^2}{1+x^2}\ dx = \int_{0.5}^{1} 1\ dx - \int_{0.5}^{1} \frac{1}{1+x^2}\ dx$

$= |x - \tan^{-1}x|$

$=(1 - \tan^{-1}1) - (0.5 - \tan^{-1}0.5)$

=0.24146- 0.03635 = 0.1783

Absolute error = $|0.1783 - 0.1782|$

= 0.0001

Relative error = $\frac{0.0001}{0.1783} = 0.00056$

M1 *for integrating &substituting the limits*
B1 *cao to 4dp*
M1 *iff modulus sign included*

B1

B1

<table>
<tr><td></td><td>Percentage error = $0.00056 \times 100 = 0.056$</td><td>B1 cao</td></tr>
<tr><td></td><td></td><td>12</td></tr>
<tr><td>3.(a)

(b)</td><td>
<table>
<tr><td>x</td><td>$10'$</td><td>$12'$</td><td>$18'$</td></tr>
<tr><td>$cosec60°$</td><td>y_1</td><td>1.1524</td><td>1.1512</td></tr>
</table>

$$\frac{1.1512-1.1524}{18'-12'} = \frac{1.1524-y_1}{12'-10'}$$
$$\therefore y_1 = 1.1528$$

<table>
<tr><td>x</td><td>$24'$</td><td>x_1</td><td>$30'$</td></tr>
<tr><td>$cosec60°$</td><td>1.1501</td><td>1.1497</td><td>1.1490</td></tr>
</table>

$$\frac{1.1490-1.1501}{30'-24'} = \frac{1.1497-1.1501}{x_1-24'}$$
$$x_1 = 26'$$
$$\therefore cosec^{-1}1.1497 = 60°26'$$
</td><td>B1 <i>for the table</i>

M1 <i>for the gradient application</i>

A1 cao to 4dp

M1 <i>for the gradient application</i>

A1 cao <i>take note the answer is not 26'</i></td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>4.</td><td>
$$1\ week = 7 \times 24 = 168\ hours$$

<table>
<tr><td>$hours$</td><td>168</td><td>336</td><td>504</td><td>672</td></tr>
<tr><td>$height\ (cm)$</td><td>4.51</td><td>4.92</td><td>5.10</td><td>5.36</td></tr>
</table>

<table>
<tr><td>$hours$</td><td>504</td><td>672</td><td>840</td></tr>
<tr><td>$height\ (cm)$</td><td>5.10</td><td>5.36</td><td>h_1</td></tr>
</table>

$$\frac{h_1-5.10}{840-504} = \frac{5.36-5.10}{672-504}$$
$$\therefore h_1 = 5.62cm$$

<table>
<tr><td>hours</td><td>168</td><td>t_1</td><td>336</td></tr>
<tr><td>Height (cm)</td><td>4.51</td><td>4.86</td><td>4.92</td></tr>
</table>

$$\frac{4.92-4.51}{336-168} = \frac{4.86-4.51}{t_1-168}$$
$$\therefore t_1 = 311.4146\ hours$$
</td><td>B1 <i>for the table</i>

M1 <i>for the gradient application</i>

A1 cao to 2dp

M1 <i>for the gradient application</i>

A1 cao
<i>Answer should be in hours</i></td></tr>
<tr><td></td><td></td><td>05</td></tr>
<tr><td>5.</td><td>
$$v = \pi r^2 h = \pi(5^2)(10) = 250\pi = 785.3982$$
$$v_{max} = \pi(5.2^2)(10.5) = 283.88\pi = 891.9609$$
$$v_{min} = \pi(4.8^2)(9.5) = 281.88\pi = 687.6318$$
$$\Delta v = \frac{1}{2}(v_{max} - v_{min})$$
$$= \frac{1}{2}(891.9609 - 687.6318)$$
$$= 102.1646$$
$$percentage\ error = \frac{\Delta v}{v} \times 100$$
</td><td>B1

B1

M1
B1</td></tr>
</table>